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Multi-Objective Optimization Model for Renewable Energy Technologies under Uncertainty: A South African Case

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Abstract

This paper presents an original multi-objective optimization model for increasing the generation capacity of renewable technologies under stochastic power outages and load shedding. The model aims to minimize the total costs of capital, and operations and maintenance (O&M), while maximizing the number of total annual jobs created in the manufacturing, installation, and operation phases of the renewable energy sources (RES). The model focuses on wind, solar photovoltaic (PV), and concentrated solar power (CSP) technologies predominantly used in South Africa. Lithium-ion batteries are used as a storage system to stockpile energy for the anticipation of load shedding. Two intervention methods, Expected Value Approach (EVA) and Scenario Chance Constrained Programming (SCCP), are applied to derive the deterministic equivalents of the stochastic constraints. A preemptive optimization approach is used to reduce the multi-objective model to single-objective solutions. The coefficient of variation is then used to compare the variation of the renewable generation from the two methods to the profile of the historical generation. The SCCP has a lower percentage error of variation than the EVA, thus validating its application when precision is critical. A sensitivity analysis is performed by varying the minimum required contribution parameter, which shows an upward trend in the total costs and annual jobs mainly influenced by increasing the capacity of CSP and PV respectively. The study uses a novel model to estimate renewable generation required to offset generation deficit, thus illustrating that increasing the nominal capacities of the RES in South Africa can mitigate the crisis of power outages.

Keywords
Renewable Energy Sources (RES), optimization, uncertainty, job opportunities, power outages

1. Introduction
Electrical energy plays a vital role in the perpetual growth and development of a country's economy. It is important to take more measures to ensure a continuous and efficient supply of electricity to support other sectors, such as production and transportation, in a country. However, the demand fluctuates because of peak periods when demand is higher and the continuous growth of electricity consumers. If the demand is higher than the supply, an imbalance is created, consequently leading to the complete collapse of the power system, which can result in nationwide power outages. The equilibrium state between the supply and demand side is of paramount concern in the operation of the grid (Wang et al. 2014). Forecasting demand and planning supply to meet the demand serves as an important management strategy.

In the year 2022 and 2023, South Africa experienced 3776 hours (157 days) and 6907 hours (288 days) of load shedding respectively. Load shedding, also known as Manual Load Reduction (MLR), is the amount of demand that has been reduced to maintain a balance between supply and demand (Eskom 2025). Load shedding is a demand-side management response to unplanned events to safeguard the power system from complete failure. Some of the negative implications of power outages include loss of earnings resulting from the cost of unserved energy, clients' loss of confidence from dissatisfaction, and decline in goods production, hence loss of revenues resulting from loss in sales. Nepal's GDP has incurred a cumulative loss of US $11 billion in nine years between 2008 and 2016 because of power outages (Timilsina and Steinbuks 2021).

Eskom is South Africa's state-owned electric company, producing 95% of the electricity used in the country and 45% in the continent (International Trade Administration 2025). Eskom is responsible for the generation, transmission, and distribution of electricity to the mining, industrial, agricultural, commercial, and residential consumers and redistributors. To satisfy the prevailing demand, the older power plants and infrastructure (coal-fired stations) are utilized at maximum capacity, which is discouraged. This deviation between the power generated to meet demand and the cold reserve capacity is under 6%. This is below the global recommended safe minimum of 15% to allow for breakdown and maintenance (South Africa: Power Outage 2008). Eskom faces the predicament of a constrained power system that will remain a problem until significant additional power generation capacity is available.

When the amount of electricity generated from the available coal-fired stations is insufficient due to unplanned power outages, Open Cycle Gas Turbines (OCGT) - diesel generators are used to maintain the energy balance (Eskom 2025). These diesel generators are expensive and pollute the environment. The challenge is to find better cost-effective and environmentally friendly sources of energy that can supplement other power sources and ease the stress on using the old coal-fired power stations. Increasing the number of Renewable Energy Sources (RES) emerges as a potential practical and sustainable solution to increase the generation capacity.

To align with Sustainable Development Goal (SDG) 7, which aims at universal access to affordable clean energy by 2030, it is important to consider the use of RES at utility-scale (Jayachandran et al. 2022). Utility-scale generation refers to the large-scale operation of power generators that are directly connected with high voltage transmission lines to provide electricity to a wide region. These sources usually include large wind farms, solar farms, hydro-electric power stations, and geothermal plants. The high-level integration of RES to the broader grid raises concerns on the reliability and stability of the power grid due to their inherent stochastic generation (Alam et al. 2020). With regards to wind and solar power plants, this characteristic is a result of the variation in wind speeds and solar irradiation. To mitigate the possibility of having an unstable power network, the uncertain generation property of RES should be accounted for in the modeling process.

1.1 Objectives

Several studies in the literature have shown different modeling techniques that are suitable to address the generation uncertainty of RES. Abdin et al. (2022) proposed a multistage adaptive robust model that considered the uncertainty of demand and renewable energy for the long-term planning of generation expansion while minimizing investment costs. Min et al. (2018) used a sample average approximation method to obtain the deterministic equivalent of a stochastic model that aimed to minimize the total costs of increasing power plant capacities. Ibrahim and Ayomoh (2022) proposed a multi-objective deterministic model to minimize the total cost of energy and greenhouse gas emissions while maximizing the number of job opportunities created in the installation and operation phases of the new power plants. There is a need to consider the problem as multi-objective to reduce costs and boost job opportunities, and as stochastic to manage the intermittent nature of RES, and unplanned power outages. Subsequently, the study presents the following research questions:

1. What is the minimum total cost that can be achieved to increase the capacity of RES required to offset the generation deficit caused by unplanned power outages?
2. What is the number of job opportunities (in the manufacturing, construction, and operation phases of RES) that can be realized by increasing RES capacities?

This paper considers developing a stochastic multi-objective model for the integration of more RES on a utility-scale in South Africa. The core objectives of the model are to minimize the total costs required to increase the generation capacity while maximizing the job creation opportunities. The key contribution of the study is an original model that estimates new nominal capacities for RES using historical generation data in South Africa. The inclusion of jobs created in the manufacturing phase of RES in the objective is also unique to this research. To the best of the authors' knowledge, there is no study similar to the formulation derived in this paper.

The rest of the paper is structured as follows: Section 2 reviews the literature on potential optimization modeling techniques for RES. Section 3 discusses the proposed mathematical model and the deterministic equivalents of the stochastic constraints. Section 4 briefly touches on data collection. Numerical results, validation, and sensitivity analysis are presented in section 5, while section 6 concludes the paper and suggests recommendations.

2. Literature Review

South Africa is estimated to have the potential to harness 6700 GW of nominal capacity for wind energy and has an average solar irradiation ranging between 4.5 to 6.6 kWh/m^2 (Jain and Jain 2017). Different stochastic mathematical models and algorithms have been developed in literature to address the uncertainty aspect of RES. The methods employed for optimization typically depend on factors such as the nature of the problem, the type of data, the degree to which the analysis and solution methodology are compatible, and the desire to employ novel or modified methods in Operations Research (OR) (Srivastava 2007). This study explored literature on mathematical models that resulted in exact formulations, which ensure global optimal solutions.

Mathematical models with random parameters and/or variables can be solved by employing a two-stage stochastic optimization approach. In two-stage stochastic programming, there are two groups of decisions and their related variables. The first stage decision variables are made before the discovery of the uncertain parameters; following this uncertainty revelation, a second series of decision variables known as recourse are made (Mavromatidis et al. 2018). Abunima et al. (2022) addressed uncertainty and fluctuation in power systems by proposing a two-stage stochastic model integrated with an artificial neural network for power prediction to minimize the cost of operation and loadshedding. Although two-stage stochastic optimization enhances robustness in decision making, it is computationally intensive as it requires the evaluation of several steps to derive the dual decomposition structure (deterministic equivalent).

Chance-constrained programming is also suitable for stochastic models where the goal is to minimize a convex objective across solutions that fulfill a system of randomly perturbed convex constraints with a probability that is close to one (Nemirovski and Shapiro 2007). Nemirovski and Shapiro (2007) note that scenario chance constrained programming's (SCCP) main advantage lies in its 'distribution free' feature whereby its application does not depend on prior knowledge of the distribution of the parameter. However, it can be computationally extensive depending on the sample size of scenarios, whereby the solution program takes time to solve. The authors mention another approximation method, the analytical chance constrained program (ACCP), which is independent of the sample size inherent in SCCP but requires that the random parameters possess 'nice' distributions. SCCP is one of the most popular methods used to solve power systems chance chance-constrained optimization problems such as demand response and economic dispatch (Geng and Xie 2019).

Focusing on objective functions, most real-world problems involve more than just a single objective as the scale of merit (Stewart 2007). If a problem is modeled with more than one objective, the model must be reduced to a sequence of single objectives to acquire practical results; examples of intervention methods are preemptive optimization and weighted sums of objectives (Rardin 1998). The weighted sums of objectives involve the combination of the multi-objective functions into a single composite function where each objective is assigned a weight. The key challenge in this approach is the impartial capturing of the relative importance of each criterion. In contrast, preemptive optimization involves the hierarchical solving of objective functions one at a time in order of importance. Once the priority objective is solved, it is used as a constraint to solve the next objective function. The main advantage of the preemptive approach is that it results in an efficient solution that cannot be improved in one objective without degrading another (Rardin 1998).

From the short review on stochastic intervention techniques and multi-objective solution methods, SCCP is selected for this study because it is less computationally intensive than the two-stage stochastic programming. It is also favored against the ACCP, because of its 'distribution free' property. The preemptive multi-objective method is preferred to reduce the problem to single objectives because the result yields an efficient solution. In the next section, the exact mathematical model is formulated.

3. Methods

3.1 Mathematical model

Implementation of additional RES to supplement the coal-fired stations is a potential solution to help mitigate the problem of unplanned power outages caused by the frequent breakdown of old coal-fired stations. Achieving this at the lowest possible cost is imperative while considering economic benefits such as job creation. A mixed integer non-linear model is proposed to determine the nominal capacity of new RES to be installed and operated with the objectives of minimizing the total cost of installation and operations and maintenance (O&M) whilst maximizing the number of job opportunities created in the process. The model proposes increasing the generation capacity of the current wind farms, solar photovoltaic (PV), and concentrated solar power (CSP) plants.

3.1.1 Definition of sets, parameters, and variables

The following sets are defined:

Let set $i \in I$ represent the set of possible RES, where:

$$I = \{1, \quad Wind\ 2, \quad PV\ 3, \quad CSP$$

Let set $s \in S$ represent the set of seasons in a year, where:

$$S = \{1, \quad Summer\ (1\ Dec\ 2022 - 28\ Feb\ 2023)\ 2, \quad Autumn\ (1\ Mar\ 2023 - 31\ May\ 2023)\ 3, \quad Winter\ (1\ Jun\ 2023 - 31\ Aug\ 2023)\ 4, \quad Spring\ (1\ Sep\ 2023 - 30\ Nov\ 2023)$$

Let set $t \in T$ represent the set of different time periods, where:

$$T = \{1, \quad hour\ 1, i.e., time\ period\ 00{:}00 - 00{:}59\ 2, \quad hour\ 2, i.e., time\ period\ 01{:}00 - 01{:}59\ \vdots\ 24, \quad hour\ 24, i.e., time\ period\ 23{:}00 - 23{:}59$$

The main assumption for the variables of the model is that the amount of energy generated at time $t \in T$ is assumed to be constant for all days in season $s \in S$, while for the parameters, the days in the season are treated as the criterion that makes the generation random at time *t*. The parameters for the model are defined below:

c_i^R	$\triangleq$	the capital cost (in \$/MW) of installing RES $i \in I$.
f_i^R	$\triangleq$	the fixed O&M cost (in \$/MW) of RES $i \in I$.
v_i^R	$\triangleq$	the variable O&M cost (in \$/MWh) of RES $i \in I$.
c^B	$\triangleq$	the capital cost (in \$/MW) of installing batteries.
f^B	$\triangleq$	the fixed O&M cost (in \$/MW) of operating batteries.
v^B	$\triangleq$	the variable O&M cost (in \$/MWh) of operating batteries
u_i	$\triangleq$	the job creation effective factor (in /MWh) in the components manufacturing stage of RES $i \in I$.
v_i	$\triangleq$	the job creation effective factor (in /MWh) in the construction/ installation stage of RES $i \in I$.
w_i	$\triangleq$	the job creation effective factor (in /MWh) in the operations stage of RES $i \in I$.
$L_{i,s,t}^L$	$\triangleq$	the historical lower half data of energy generated (in MWh) from RES $i \in I$, in season $s \in S$, at time $t \in T$. The lower half consists of data ranging from the minimum amount ever produced at time *t* up to the *median-1* index value.
$L_{i,s,t}^U$	$\triangleq$	the historical upper half of energy generated (in MWh) from RES $i \in I$, in season $s \in S$, at time $t \in T$. The upper half consists of data ranging from the *median+1* index value up to the maximum amount ever produced at time *t*.
$G_{s,t}$	$\triangleq$	the energy generation deficit (in MWh) caused by unplanned outages in season $s \in S$ at time $t \in T$. This is the amount of energy that a power plant that is unexpectedly off is capable of supplying.
r_i	$\triangleq$	the minimum percentage generation requirement that has been set for RES $i \in I$.
k_i	$\triangleq$	the current capacity factor of RES $i \in I$.
$\tilde{a}_{s,t}$	$\triangleq$	the probability of load shedding happening in season $s \in S$ at time $t \in T$.
$G_{s,t}^{Pk}$	$\triangleq$	the given peak generation deficit (in MWh) in season $s \in S$ at time $t \in T$.
M_i	$\triangleq$	the current capacity of RES $i \in I$.
d_s	$\triangleq$	the number of days in season $s \in S$.
η^C	$\triangleq$	the round-trip efficiency of the battery when charging.

η^D ≜ the round-trip efficiency of the battery when discharging.

The following variables were defined for the model.

$x_{i,s,t}^L$ ≜ the amount of energy generated (in MWh) from RES $i \in I$ in season $s \in S$ at time $t \in T$ that can be used to offset generation deficit.
$x_{i,s,t}^B$ ≜ the amount of energy generated (in MWh) from RES $i \in I$ in season $s \in S$ at time $t \in T$ and sent to the batteries.
$b_{i,s,t}^L$ ≜ the amount of energy (in MWh) from battery connected to source $i \in I$ that can be used to offset generation deficit in season $s \in S$ at time $t \in T$.
$E_{i,s,t}$ ≜ the state of charge (in MWh) of the batteries connected to source $i \in I$ in season $s \in S$ at time $t \in T$.
N_i ≜ The additional nominal capacity (in MW) of RES $i \in I$ required to offset the generation deficit.
H ≜ the number of batteries needed to store excess energy from the RES.
$y_{s,t}$ ≜ binary variable to allow CSP technology the freedom to choose when to generate energy in season $s \in S$ at time $t \in T$.

3.1.2 The objective functions

The proposed model has two objective functions represented by Equations (1) and (2). The two objectives are formulated to minimize the installation, and O&M costs while maximizing job creation in the manufacturing, installation, and operation phases of the RES.

$$z_1 = \sum_{i=1}^{I} N_i(c_i^R + f_i^R) + \sum_{i=1}^{I} v_i^R \sum_{s=1}^{S} \sum_{t=1}^{T} \left(x_{i,s,t}^L + x_{i,s,t}^B\right) d_s + 100H(c^B + f^B) + v^B \sum_{i=1}^{I} \sum_{s=1}^{S} \sum_{t=1}^{T} b_{i,s,t}^L d_s \tag{1}$$

$$z_2 = \sum_{i=1}^{I} \left[(u_i + v_i + w_i) \sum_{s=1}^{S} \sum_{t=1}^{T} \left(x_{i,s,t}^L + x_{i,s,t}^B\right) d_s\right] \tag{2}$$

The first term in (1) represents the total installation costs, and fixed O&M costs for RES. The second term captures the variable O&M costs. The third term includes the installation and fixed O&M costs for battery units with a power capacity of 100 MW, while the fourth term accounts for the variable O&M costs of the batteries. These costs are calculated over one year. The second objective function, Equation (2), represents the total annual jobs created during the manufacturing of RES components, the installation of the sources, and their operation.

3.1.3 Energy generation deficit satisfaction constraint

$$\sum_{i=1}^{I} x_{i,s,t}^L + \eta^D \sum_{i=1}^{I} b_{i,s,t}^L \geq \tilde{G}_{s,t} \qquad \forall\ s \,\epsilon\, S, t \,\epsilon\, T \tag{3}$$

Equation (3) is a constraint that ensures sufficient energy is generated during each time $t \,\epsilon\, \boldsymbol{T}$ in season $s \,\epsilon\, \boldsymbol{S}$. The generation deficit can be offset with the amount of energy coming directly from the RES and/or from the batteries.

3.1.4 Battery state of charge constraint

$$E_{i,s,1} = E_0 + \eta^C x_{i,s,1}^B - b_{i,s,1}^L \qquad \forall\ i \in I, s \,\epsilon\, S \tag{4}$$

$$E_{i,s,t} = E_{i,s,t-1} + \eta^C x_{i,s,t}^B - b_{i,s,t}^L \qquad \forall\ i \in I, s \,\epsilon\, S, t \in T | t > 1 \tag{5}$$

The state of charge of the battery storage system connected to the different sources for each time in a particular season is determined by (4) and (5). These are inventory-type constraints where the amount of power in the batteries from the previous period determines the current period's state of charge. Given that the model's main assumption is that the amount of energy generated at time *t* is constant in all days of season *s*, the system does not account for the state of charge rolling over from time *t = 24* from season *s* to time *t = 1* in season *s+1*.

3.1.5 Energy generation limits constraints

$$(x^L_{i,s,t} + x^B_{i,s,t})M_i \geq \tilde{L}^L_{i,s,t}N_i \qquad \forall\ i \in I | i < 3, s \epsilon S, t \in T \qquad (6)$$
$$(x^L_{3,s,t} + x^B_{3,s,t})M_3 \geq \tilde{L}^L_{3,s,t}N_3 y_{s,t} \qquad \forall\ s \epsilon S, t \in T \qquad (7)$$
$$(x^L_{i,s,t} + x^B_{i,s,t})M_i \leq \tilde{L}^U_{i,s,t}N_i \qquad \forall\ i \in I, s \epsilon S, t \in T \qquad (8)$$
$$N_i \geq x^L_{i,s,t} + x^B_{i,s,t} \qquad \forall\ i \in I, s \epsilon S, t \in T \qquad (9)$$

Equations (6) – (8) together ensure that the amount of energy generated to offset the generation deficit follows the uncertain profile of the RES. This prevents the RES from generating energy at a time when the energy source is known to be unavailable. Equations (6) and (7) ensure that the new amount generated from the RES does not fall below the normalized minimum historical generation, while Equation (8) ensures that the amount generated does not exceed the normalized maximum historical generation. Given that CSP technology could have in-built storage, the minimum generation requirement is multiplied by a binary variable to give the source more freedom. The assumption in these constraints is that the new generation will have the same average wind levels/ speed and solar irradiation for the wind source and solar sources, respectively, as the current power plants, but with increased resources to generate more energy. It is known that the generating capacity at different times is usually less than or equal to the nameplate capacity of these sources; Equation (9) restricts the new generation at any time from exceeding the nameplate capacity of the source.

3.1.6 Energy reserve constraints

$$\sum_{i=1}^{I} E_{i,s,t} \geq \tilde{a}_{s,t} G^{Pk}_{s,t} / \eta^D \qquad \forall\ s \epsilon S, t \in T \qquad (10)$$
$$\sum_{i=1}^{I} E_{i,s,t} \leq 80H \qquad \forall\ s \epsilon S, t \in T \qquad (11)$$

Equation (10) establishes the minimum energy to be stored as reserve to provide backup in the event of load shedding. The amount of load curtailed is estimated by the product of the probability of load-shedding and peak generation deficit at time *t*. Equation (11) determines the number of battery units to install while also ensuring that the battery charge does not exceed 80% since each unit has a power capacity of 100 MW.

3.1.7 Percentage requirement constraint

$$\sum_{s=1}^{S} \sum_{t=1}^{T} (x^L_{i,s,t} + x^B_{i,s,t}) d_s \geq r_i \left(\sum_{i=1}^{I} \sum_{s=1}^{S} \sum_{t=1}^{T} (x^L_{i,s,t} + x^B_{i,s,t}) d_s \right) \qquad \forall\ i \in I \qquad (12)$$

The different RES are limited in abundance depending on location and other factors such as cost. It is therefore prudent to have a constraint that requires each source to meet a certain proportion of the total renewable generation. These targets can function as policies, such as renewable portfolio standards (RPS). Equation (12) ensures that the generation targets of the various RES are met.

3.1.8 Capacity factor constraint

$$k_i \geq \frac{\sum_{s=1}^{S} \sum_{t=1}^{T} ((x^L_{i,s,t} + x^B_{i,s,t}) d_s}{N_i * 365 * 24} \qquad \forall\ i \in I \qquad (13)$$

Given the uncertain characteristics of RES, Equation (13) ensures that the new generation capacity factor does not exceed the current (historical) capacity factors.

3.1.9 Non-negativity and integrality constraints

$$x_{i,s,t}^{L}, x_{i,s,t}^{B}, b_{i,s,t}^{L}, E_{i,s,t} \geq 0 \qquad \forall\ i \in I, s \in S, t \in T \qquad (14)$$

$$N_i \geq 0 \qquad \forall\ i \in I \qquad (15)$$

$$y_{s,t} \epsilon \{0, 1\} \qquad \forall\ s \in S, t \in T \qquad (16)$$

$$H \geq 0, and\ integer \qquad (17)$$

Constraints (14) and (15) ensure that all variables are non-negative. Constraint (16) ensures that y is a binary variable, while (17) ensures that the number of batteries is a positive integer value.

The next subsection discusses the intervention techniques used to derive the deterministic equivalents of the stochastic constraints.

3.2 Stochastic intervention techniques

3.2.1 Expected value approach (EVA)

In the expected value approach, the deterministic equivalent of the stochastic model is simply derived by substituting the random parameters with their expected values obtained from computing the mean of the data. The original model has five stochastic constraints, i.e., (3), (6), (7), (8), and (10), whose derived deterministic equivalents are shown in (18), (19), (20), (21) and (22) respectively.

$$\sum_{i=1}^{I} x_{i,s,t}^{L} + \eta^{D} \sum_{i=1}^{I} b_{i,s,t}^{L} \geq E(\tilde{G}_{s,t}) \qquad \forall\ s \in S, t \in T \qquad (18)$$

$$\left(x_{i,s,t}^{L} + x_{i,s,t}^{B}\right) M_i \geq E(\tilde{L}_{i,s,t}^{L}) N_i \qquad \forall\ i \in I | i < 3, s \in S, t \in T \qquad (19)$$

$$\left(x_{3,s,t}^{L} + x_{3,s,t}^{B}\right) M_3 \geq E(\tilde{L}_{3,s,t}^{L}) N_3 y_{s,t} \qquad \forall\ s \in S, t \in T \qquad (20)$$

$$\left(x_{i,s,t}^{L} + x_{i,s,t}^{B}\right) M_i \leq E(\tilde{L}_{i,s,t}^{U}) N_i \qquad \forall\ i \in I, s \in S, t \in T \qquad (21)$$

$$\sum_{i=1}^{I} E_{i,s,t} \geq E(\tilde{a}_{s,t}) G_{s,t}^{Pk} / \eta^{D} \qquad \forall\ s \in S, t \in T \qquad (22)$$

The EVA is compared to the SCCP approach discussed in the following subsection.

3.2.2 Scenario chance constrained programming (SCCP)

In SCCP, the uncertainty in the parameter is substituted with a set of finite scenarios. These scenarios are usually generated by sampling from historical data or a known distribution. According to Nemirovski and Shapiro (2007), given the following minimization problem (23):

$$f(x) \quad subject\ to \quad Prob(F(x, \xi) \leq 0) \geq 1 - \alpha \qquad (23)$$

where ξ is the random vector and α is the calculated risk level from a given confidence level. The derived deterministic equivalent of (23) is modeled as (24):

$$f(x) \quad subject\ to \quad Prob(F(x, \xi^{v}) \leq 0) \geq 1 - \alpha \qquad \forall\ v \in \{1, \dots, N\} \qquad (24)$$

The size of the sample is mainly determined by the confidence level required to satisfy the constraint and the decision maker's chosen reliability level. Equation (25) is used to estimate the sampling size for the scenarios.

$$Q \geq Q^{*} = [2n\alpha^{-1} ln(12/\alpha) + 2\alpha^{-1} ln(2/\delta) + 2n] \qquad (25)$$

where Q is the sample size, n is the number of random variables in the constraint, α is the risk level of a given confidence level, and δ is the chosen reliability level (Nemirovski and Shapiro 2007).

This solution approach is tractable since the uncertain constraints are reduced to a finite set of deterministic values. However, given the linear relationship of the sample size Q and the risk tolerance α, higher confidence levels result in increased computational time. It is also important to note that the solution is dependent on the quality of the sample scenarios generated.

This intervention technique leads to the following deterministic equivalent constraints for (3), (6), (7), (8), and (10) respectively:

$$\sum_{i=1}^{I} x_{i,s,t}^{L} + \eta^{D} \sum_{i=1}^{I} b_{i,s,t}^{L} \geq G_{s,t,v} \qquad \forall\ s \epsilon S, t \epsilon T, \qquad v \epsilon \{1,..,N\} \tag{26}$$

$$\left(x_{i,s,t}^{L} + x_{i,s,t}^{B}\right) M_i \geq L_{i,s,t,v}^{L} N_i \qquad \forall\ i \in I | i < 3, s \epsilon S, t \in T, v \epsilon \{1,\ldots,N\} \tag{27}$$

$$\left(x_{3,s,t}^{L} + x_{3,s,t}^{B}\right) M_3 \geq L_{3,s,t,v}^{L} N_3 y_{s,t} \qquad \forall\ s \epsilon S, t \in T, v \epsilon \{1,\ldots,N\} \tag{28}$$

$$\left(x_{i,s,t}^{L} + x_{i,s,t}^{B}\right) M_i \leq L_{i,s,t,v}^{U} N_i \qquad \forall\ i \in I, s \epsilon S, t \in T, v \epsilon \{1,\ldots,N\} \tag{29}$$

$$\sum_{i=1}^{I} E_{i,s,t} \ \geq\ a_{s,t,v} G_{s,t}^{Pk} / \eta^{D} \qquad \forall\ s \epsilon S, t \in T, v \epsilon \{1,\ldots,N\} \tag{30}$$

The next section describes where the parameters data required for the solution of the model were obtained.

4. Data Collection

The parameter values in the model were acquired from secondary data. The costs and job factors data were obtained from published articles and reports (IRENA 2025; Short et al. 2009; Maia 2024; Rutovitz et al. 2020). These data vary with different sources depending on the region or site where the study was conducted. The unplanned power outages (generation deficit), historical generation, current capacity, capacity factor, peak generation deficit, and probability of load-shedding parameters were obtained/ derived from Eskom's data portal (Eskom 2025). The initial battery state of charge, E_0, the charging round-trip efficiency, η^C, and the discharge round-trip efficiency, η^D, are assumed to be 0, 0.9524, and 0.95 respectively. Table 1 shows the cost and job factor parameters used for the solutions. Wei et al. (2010) described a methodology to convert the job factor parameter units to /MWh.

Table 1. Cost and job factor parameter values

Source	Cost parameter			Job factor phase parameter		
	Overnight capital ($/MW)	Fixed O&M ($/MW)	Variable O&M ($/MWh)	Manufacturing (job-yrs/MW)	Installation (job-yrs/MW)	O&M (jobs/MW)
Wind	1160000	10950	4.41	4.5	1.5	0.5
PV	758000	7500	0	16.8	7	0.7
CSP	6589000	44570	0.1	14.4	21.6	0.54
Battery	325000	4700	5	6.6	4.7	1.2

The data was used in Lingo optimization solution software to obtain global optimal results, which are discussed below.

5. Results and Discussion

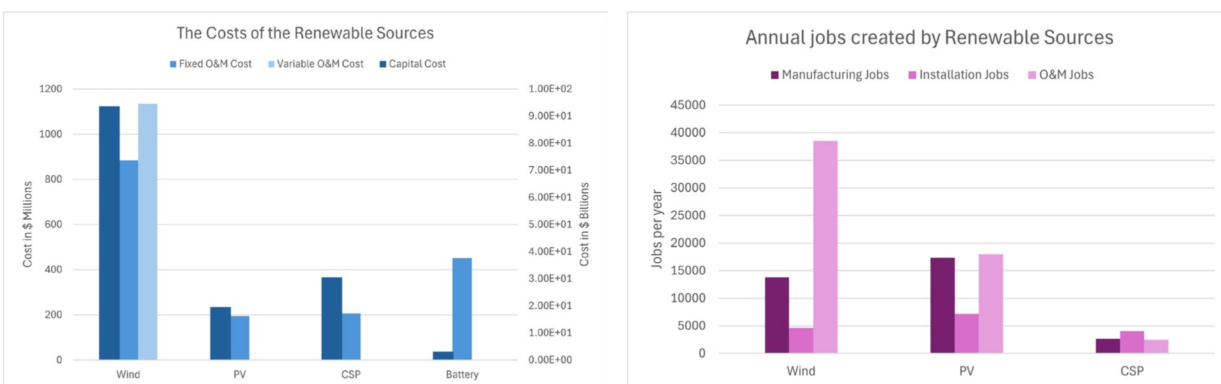
5.1 Results: EVA vs SCCP

Preemptive multi-objective approach was used to solve the model with the cost as the priority objective. The total cost was prioritized over the job creation objective since it is a primary inhibitor to RES development. The optimum total cost value was then used as an upper limit to constrain the cost objective when solving for the optimum job creation opportunities. Figures 1 and 2 show the global optimum solutions of the objective values for the EVA and SCCP intervention techniques, respectively. The capital cost (dark-blue column) is plotted on the secondary axis, while the fixed and variable O&M costs (blue and light-blue columns) are plotted on the primary axis. The EVA yielded a total cost of \$77 bn and 50,163 total annual jobs, while the SCCP, with a chosen confidence level of 90% and 0.01 reliability level, yielded \$150 bn and 108,630 total annual jobs.

The EVA yielded a cheaper cost, and lower annual jobs created than the SCCP approach. This is explained by the difference in the results of the nominal capacities of the RES for the two methods as shown in Table 2. Given the conservative nature of the SCCP, the maximum sample value for each time t in season s of the generation deficit parameter is used in the energy generation deficit satisfaction constraint in (26). These values are greater than the corresponding averages in constraint (18), thus necessitating higher nominal capacities for the RES. Assuming that the lifetime for each of the RES is 25 years, the total costs (overnight capital cost and O&M costs over the lifetime), excluding the battery storage system costs, are \$104 bn and \$205 bn, while the total jobs are 1.25 million and 2.72 million from the EVA and SCCP methods respectively. This implies that each job created using the EVA method costs \$82,923, but each job created using the SCCP method costs \$75,301. Therefore, considering the number of employment opportunities, the SCCP approach is less expensive. Other global optimal results of the new RES generation are shown in Table 2.

(a) Objective 1: EVA costs (b) Objective 2: EVA annual jobs created

Figure 1. Global optimal objective solutions for EVA intervention method



(a) Objective 1: SCCP costs (b) Objective 2: SCCP annual jobs created

Figure 2. Global optimal objective solutions for SCCP intervention method

Table 2. Global optimum results

Approach	Nominal Capacity (GW)			Total State of Charge (MWh)		Mean Generation (MWh)			No. of Batteries
	Wind	PV	CSP	Minimum	Maximum	Wind	PV	CSP	
EVA	44.1	11.9	2.14	977.8	4880.0	13.6	3.13	0.70	61
SCCP	80.7	25.8	4.63	4132.6	7680.0	29.4	6.79	1.51	96

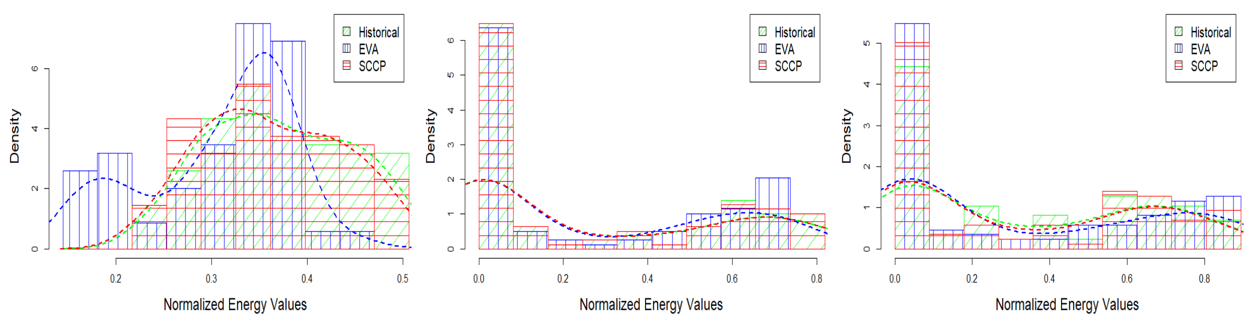
From Table 2, both solution approaches recommend generating more from the Wind power plants than the other sources despite solar PV having the lowest overnight capital cost and O&M costs as shown in Table 1. The model recommends Wind power plants to have the highest nominal capacity and mean generation because wind has the lowest variability and is more available than the other sources, as evidenced in Figure 3. Wind RES can be used to address the generation deficit when the solar-powered sources are not available to generate.

In the next section, the generation distributions of the new RES are compared to their historical (current) profile to validate the models' accuracy.

5.2 Validation: EVA vs SCCP

The core variable that drives all other variables is the amount generated from RES i at time t. This variable determines the state of charge of the battery system and the nominal capacities of the RES. Therefore, it is crucial to validate the models' recommended results with known generation profiles for the different technologies. Validating solutions underscores the significance of the accuracy levels of different approaches, hence providing insights to the decision makers. Figure 3 shows the overlaid generation distributions for the three different sources. The green hatched pattern represents the historical distribution, the red hatched pattern represents the SCCP distribution, and the blue hatched pattern represents the EVA distribution.

Looking at Figure 3, it can be deduced that for all sources, the SCCP distribution (in red) tracks the historical distribution (in green) more closely than the EVA distribution (in blue). Visual validation, therefore, confirms that the SCCP approximates the uncertainty of RES generation better than the EVA.



(a) Wind generation (b) PV generation (c) CSP generation

Figure 3. Visual validation of the distribution of the RES

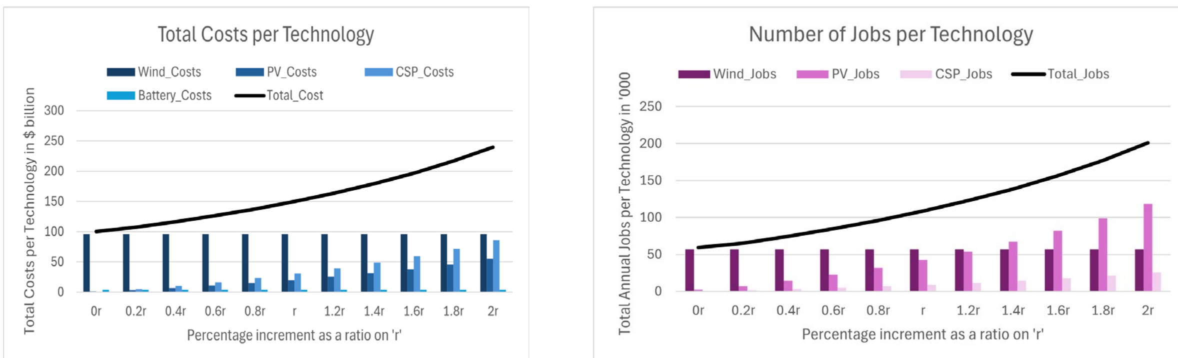
The mean values for the historical, EVA, and SCCP generations differ greatly. The coefficient of variation is a good statistical measure to compare distributions obtained with different units or means (Abdi 2010). Since the three distributions have different means, the coefficient of variation is also used for comparisons. For the wind generation, the coefficient of variation for the historical, EVA, and SCCP profiles is 20.01%, 25.02%, and 20.11%, respectively. For the PV generation, the coefficient of variation for the historical, EVA, and SCCP is 120.53%, 114.30%, and 120.29%, respectively. For the CSP generation, the coefficient of variation is 90.80%, 104.44%, and 97.05%, respectively. As expected, solar PV and CSP have high coefficients of variation given their high relative variability. Also, the historical generation coefficients of variation for all three sources are closer in value to the SCCP generation than the EVA generation. This further validates SCCP as superior in accuracy to EVA when estimating the uncertainty of RES generation. However, the EVA method is an ideal option for budget-constrained projects where precision is of less significance.

In the next section, a sensitivity analysis was performed by varying the minimum percentage requirement parameter.

5.3 Sensitivity analysis on SCCP approach

Sensitivity analysis helps decision-makers understand how varying input parameters influence the optimal solution (Razavi et al. 2021). In this study, the minimum percentage requirement parameter, r_i, was selected as the input parameter to tune and analyze how the optimal solution changes within the feasible space. This parameter determines the weights of the RES. The initial minimum requirement for wind, solar PV, and CSP was 27%, 18%, and 4%, respectively. This parameter was tuned in ratio decrements of 0.2 to 1 where the starting percentage requirement for each of the sources became 0, and increased in steps of 0.2 to 1 where the percentage requirement reached 54%, 36%, and 8% for wind, solar PV, and CSP, respectively. Figure 4 shows the effect on the total costs and number of jobs created.

As expected, decreasing the percentage minimum requirement results in a decrease in the total costs and total annual jobs created, while increasing the percentage minimum requirement leads to an increase in the two objectives. From Figure 4, it can be deduced that the increasing total costs are influenced mostly by the increasing requirement for CSP contribution. This is confirmed by the high capital and fixed O&M costs associated with CSP power plants. The upward trend of the total annual jobs seems to be driven by the increasing requirement for PV contribution. This is explained by the fact that PV has the highest aggregate job factor compared to the other sources.



(a) Sensitivity results on costs (b) Sensitivity results on jobs created

Figure 4. Sensitivity analysis results on SCCP

6. Conclusion

Operations research is an important area of research for modeling uncertainties in energy systems. This paper formulated a mixed integer non-linear stochastic multi-objective model to minimize the total costs of capital, fixed and variable O&M, and maximize the number of job opportunities. The uncertainties in RES and unplanned power outages were approximated with their expected values using EVA, and scenarios using SCCP. The EVA yielded lower total costs and job opportunities than the SCCP approach. However, validating EVA and SCCP RES generation against the historical generation showed that the SCCP method approximated the uncertainty better than the EVA. This is important for decision-makers since the correct generation characteristic will ensure a reliable and stable grid when integrating RES. Despite this, the EVA method can be considered in instances with financial limitations. If budget is a concern, decision-makers can use the EVA recommendations, but if a stable grid is more important, the SCCP recommendations should be used. Sensitivity analysis revealed that there is an existing direct proportional relationship between the objectives, the nominal capacities, and the amount of energy generated.

The model can be improved by adding an annual budget as a constraint, thus having an expansion plan spanning several years/ periods. This is quite realistic since the overnight expansion of RES is impracticable for a developing country.

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