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Optimal Operation of Vending Machines

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Abstract

In this study, we address the operational policy problem faced by a seller managing a portfolio of vending machines. The objective of our study is to develop a mathematical model that optimally manages the operations of multi-item vending machines distributed across various locations to maximize profit. Our proposed model takes a comprehensive approach by simultaneously considering several key aspects such as pricing, inventory management, capacity planning, and route optimization while addressing demand uncertainty using a scenario-based approach. In particular, we aim to answer several key questions: which items should be available at each location and how many towers should be allocated for each item; what the optimal inventory level of these items should be at the beginning of each period;

whether any transfers between locations are required during each time period; and what the price of each item should be at each location and for each period. The objective of our modeling approach is to determine a pricing policy that effectively balances demand and inventory. Numerical experiments are conducted to evaluate the robustness and efficiency of the model's solutions.

Keywords
Vending Machine; Inventory Management; Dynamic Pricing; Routing; Mathematical Programming.

1. Introduction
Over the past few decades, technological advancements across fields such as operations, marketing, and logistics have significantly intensified competition across many markets. To survive in this environment, players must act strategically to improve their supply chain management capabilities. One such strategy is expanding market share through the use of vending machines, which has become common in sectors such as retail, healthcare, education, entertainment, and transportation. Retailers, for example, use vending machines to offer products in high-traffic locations, while healthcare providers offer medical supplies in hospitals. Educational institutions, entertainment venues, and transportation hubs also use vending machines to improve convenience and customer service. This practice helps businesses reach more customers and streamline product distribution across various locations, increasing their competitive edge.

With the help of latest advances in technology, companies can now manage their vending machine operations more effectively. For example, parameters such as stock levels and product conditions within vending machines can be closely monitored and optimized with the latest technology. Additionally, widespread internet use by both consumers and businesses enables customers to easily access information about prices for substitute products. The increasing use of intelligent technologies and real-time data-driven applications has considerably changed how vending machine operations area managed. Vendors can no longer rely on pre-scheduled refill routines. Modern vending machines require real-time inventory tracking, dynamic pricing strategies, and effective replenishment planning to succeed in fast-moving markets. Therefore, flexible and dynamic pricing has become crucial for companies to remain competitive in the market. To adjust prices optimally over time, businesses must consider factors such as operating costs, supply availability, future demand, and customers' perceived value.

The problem of managing vending machines across multiple locations with multiple products remains a complex challenge for many companies. Planners and operators must make decisions about pricing, inventory levels, available machine space, and product delivery routes simultaneously, while considering the uncertainty in demand. Much of the existing research focuses on just one or two of these factors, without considering the impact of coordinating all these features with a holistic perspective.

In this study, our main ambition is to develop an integrative mathematical model designed to optimize the performance of multi-item vending machines across various locations. Our model considers dynamic pricing, inventory management, capacity planning, and route optimization, while addressing demand uncertainty using a scenario-based approach. Our numerical experiments demonstrate that our methodology delivers effective solutions.

The remainder of this paper is organized as follows. Section 2 reviews the related literature and defines the scope of our study. Section 3 presents our proposed mathematical model. Section 4 discusses the numerical experiments and compares our approach with alternative methods. Finally, Section 5 concludes the paper and suggests future research directions.

2. Scope and Relevant Studies
In this section, we review the related literature on vending machine operations with a focus on the challenges of inventory management, pricing, and replenishment under uncertainty. Several studies have addressed specific aspects of vending machine operations, but few provide an integrated approach that simultaneously considers pricing, inventory, and routing decisions.

Elmaghraby and Keskinocak (2003) highlight the influence of market characteristics on a retailer's dynamic pricing problem, and they emphasize the factors such as replenishment opportunities during the planning horizon, demand dependency over time, product life, and customer knowledge significantly impact inventory decisions and price optimization. Additionally, customer behavior, seasonality in demand, and business rules play key roles in pricing and inventory routing. In their study, products are classified as replenishable, time-independent in demand, and uncorrelated in terms of substitutability or complementarity. They also review literature on pricing and procurement problems with replenishable inventory, categorizing them based on demand uncertainty, production or procurement capacity, and cost function characteristics. The problems most closely related to our study involve uncertain (stochastic) demand, convex cost functions, and uncapacitated procurement or production. Chen and Simchi-Levi (2012) further classify these problems as multi-period models, which are also relevant to the current research.

Despite the recent focus on vending machine operations, there are still significant challenges in this field. For instance, some studies, such as those by Park and Park (2015), Park and Yoo (2013), Poon et al. (2010), and Solano et al. (2017), assume real-time inventory visibility but do not account for demand stochasticity. Other studies, such as those by Rusdiansyah and Tsao (2005), focus primarily on vehicle routing problems, aiming to minimize inventory holding and travel costs without integrating pricing and inventory decisions. Lin et al. (2011), on the other hand, propose a product recommendation system for vending machines, using a hybrid approach combining meta-heuristics, classification, and clustering techniques. However, they do not address the optimal allocation of space in the machines. Park and Yoo (2012) introduce a heuristic for inventory management with product substitution but do not account for demand variability in their model. Their proposed developed heuristic is built on a decoupled approach. A mathematical model finds the number of product storage compartments and replenishment threshold for each vending machine and then a savings algorithm is used to find vehicle routes for replenishment of the vending machines. However this study does not incorporate the stochastic nature of sales.

Grzybowska et al. (2020) suggest a simulation-optimization approach to optimize product allocation in vending machines to minimize restocking while maximizing revenue. In a more recent study, Rios and Vera (2023) introduce an integrated model for dynamic pricing and inventory replenishment across a retail chain, accounting for substitution effects and uncertain demand. Their approach uses stochastic optimization to improve profitability. Sadeghi et al. (2014) introduce a bi-objective vendor managed inventory model aiming to find the order size, the replenishment frequency, the optimal traveling tour from the vendor to retailers, and the number of machines in order to minimize the total cost and maximize the production system reliability. They use multi-objective genetic algorithms to solve the problem. The mentioned models focuses on the routing problem, and do not provide a comprehensive approach for the operation of vending machines.

In an uncertain demand environment, the planning decision is expected to perform well across possible future realizations. A powerful approach for representing uncertainty in planning is the use of scenarios, as proposed by Kang and Lansey (2013), to model probabilistic parameters in the system. This method allows for the development of a robust solution that accounts for various potential future scenarios, each associated with a specific probability of occurrence. Scenario-based models are commonly used in decision problems aimed at generating schedules for future application, helping to manage uncertainty and reduce the search space to a tractable size. Recent literature providing examples of scenario-based approaches includes Gao and Cao (2020), Wu and Hifi (2020), and Merdanoglu et al. (2020).

In another study, Li et al. (2023) present a scenario-based distributionally robust optimization model for the stochastic inventory routing problem under demand uncertainty. This model determines routing choices, replenishment quantities, and timing. The authors propose an algorithmic framework that integrates Tabu Search and column generation algorithms to efficiently solve large-scale instances. Their results demonstrate that scenario-based models outperform traditional logistic models by better addressing demand variability and improving operational performance under uncertain conditions.

Our study presents a scenario-based optimization approach to simultaneously determine the length of the planning horizon, space allocation in vending machines, initial inventory levels, transfer requirements between pairs of locations (where the machines and supply centers are situated), and item prices. To the best of our knowledge, this problem and the proposed mathematical model carry unique features, and no prior work in the relevant literature defines this problem.

3. Mathematical Model

The problem we consider in this study is based on the following assumptions: (*i*) the demands for products are independent and stochastic, (*ii*) each vending machine can contain multiple towers, with each tower holding only one type of item, (*iii*) dynamic pricing is allowed, (*iv*) backlogging is not permitted, (*v*) customers make decisions based on the instantaneous valuation of products, without consideration of potential future price changes, (*vi*) the total cost includes procurement costs, holding costs, and delivery costs, (*vii*) replenishment can occur either from supply centers or from other machines, and (*viii*) lead time is assumed to be zero for supply centers.

Based on these assumptions, our goal is to determine the optimal solution to the following questions: (*i*) which items should be available at each location, and how many towers should be allocated for each item, (*ii*) what should the optimal inventory level of these items be at the beginning of each period, (*iii*) is any transfer between locations required during each time period, and (*iv*) what should the price of each item be at each location and for each time period? The objective is to find a pricing policy that balances demand and inventory. The decisions regarding product combinations and replenishment for each location are made simultaneously. This section introduces the required notation and presents the proposed mathematical model to solve the problem described.

Below, we introduce the notation, sets, indices, parameters, decision variables and the formulation for the proposed mathematical model.

3.1 Sets

$P = \{1, \ldots, p, \ldots, |P|\}$: set of products,
$R_p = \{1, \ldots, r_p, \ldots |R_p|\}$: set of price indices for product p,
$V = \{1, \ldots, v, \ldots, |V|\}$: set of vending machine locations,
$C = \{1, \ldots, c, \ldots, |C|\}$: set of supply center locations,
$L = \{1, \ldots, l, \ldots, |L|\}$: set of locations $(V \cup C)$,
$T = \{1, \ldots, t, \ldots, |T|\}$: set of periods,
$T'_{ll'} \subset T$: subset of periods when items are allowed to be transferred from location l to location l' (where l' is an alias for l),
$S = \{1, \ldots, s, \ldots, |S|\}$: set of scenarios.

3.2 Parameters

$d_{p,r_p,l,t,s}$: demand forecast for product p at location l, at period t when its price is r_p in scenario s,
c_p: procurement cost of product p,
h_p: holding cost of product p for one period,
$c^t_{l,l'}$: transfer cost from location l to l',
$l^t_{l,l'}$: required length of periods for transferring a product from location l to l',
w_l: number of available towers at location l,
y_p: tower capacity for product p,
m_{r_p}: monetary value of price r_p.

3.3 Decision Variables

$a_{p,r_p,l,t}$: 1 if product p is available for sale at location l, at period t for price r_p; 0 o.w.,
$b_{p,l,t} \in I^+$: number of towers allocated to product p at location l at period t,
$n_{p,l,t,s} \in I^+$: stock level of product p at location l at the beginning of period t in scenario s,
$r_{p,l,l',t,s} \in I^+$: amount of product p transferred from location l to location l'at period t in scenario s,
$z_{l,l',t,s}$: 1 if any item is transferred from location l to location l'at period t in scenario; 0 o.w.,
$u_{p,r_p,l,t,s} \in I^+$: amount of product p sold at location l at period t for price r_p in scenario s.

3.4 Objective Function

The objective function maximizes the average profit over scenarios. The function includes generated revenue, procurement cost, holding cost and delivery cost components. Total profit is divided by the total number of scenarios to find the average profit.

$$max \frac{1}{|S|}\left(\begin{array}{c}\sum_{s}\sum_{p}\sum_{r_p}\sum_{l:l\in V}\sum_{t} m_{r_p}u_{p,r_p,l,t,s} - \sum_{s}\sum_{p}\sum_{l:l\in V} c_p\left(n_{p,l,0,s} - n_{p,l,|T|,s}\right) \\ -\sum_{s}\sum_{p}\sum_{l:l\in V}\sum_{t} h_p n_{p,l,t,s} - \sum_{s}\sum_{t}\sum_{l}\sum_{l'} c_{l,l'}^{t} z_{l,l',t,s}\end{array}\right)$$

3.5 Constraints

(1) At a certain location and period, a product can have only one price. If there is no product at location l at time period t, no price is required.

$\sum_{r_p} a_{p,r_p,l,t} \leq 1$ $\qquad\qquad \forall p, l, t$

(2) Inventory level of a product at location l is found by considering the previous inventory level, amount of product transferred to/from the location and the amount of product sold at the location during the previous time period. Constraint provides the balance of inventory at each location throughout the time horizon.

$n_{p,l,t,s} = n_{p,l,t-1,s} - \sum_{l'} r_{p,l,l',t-l_{l,l'}^{t},s} + \sum_{l'} r_{p,l',l,t,s} - \sum_{r_p} u_{p,r_p,l,t,s}$ $\qquad\qquad \forall p, l, t > 0, s$

$\left(n_{p,l\in V,t=0,s} \text{ is given and } n_{p,l=SC,t=0,s} = M, \text{where } M \text{ is a big number}\right)$

(3) To transport a product between locations, a transportation mean must be present at the origin location l at the appropriate period. $T'_{u'}$, is the set of periods when items can be transferred from origin l to destination l'. So when t is not $T'_{u'}$, no transportation can occur between locations l and l'.

$z_{l,l',t,s} = 0$ $\qquad\qquad \forall l, l', t \in T \setminus T'_{u',s}$

(4) Delivery cost covered in the objective function is the multiplication of the delivery cost and the binary value $z_{l,l',t,s}$ that represents whether the transportation between locations l and l'occurs or not. If there is any transportation from location l to l', then the objective function must include the corresponding cost.

$z_{l,l',t,s}M \geq \sum_p r_{p,l,l',t,s}$ $\qquad\qquad \forall l, l', t, s$ (M is a big number)

(5) Amount of product p sold at location l at period t must be less than both its inventory level and its demand at location l at period t.

$u_{p,r_p,l,t,s} \leq a_{p,r_p,l,t} d_{p,r_p,l,t,s}$ $\qquad\qquad \forall p, r_p, l, t, s$

$\sum_{r_p} u_{p,r_p,l,t,s} \leq n_{p,l,t,s}$ $\qquad\qquad \forall p, l, t, s$

(6) Since a tower has a certain capacity for each product, the amount of product p at location l cannot exceed the total capacity of allocated towers for product p at location l. Moreover, any tower must not be empty.

$n_{p,l,t,s} > y_p\left(b_{p,l,t} - 1\right)$ $\qquad\qquad \forall p, l \in V, t, s$

$n_{p,l,t,s} \leq y_p b_{p,l,t}$ $\qquad\qquad \forall p, l \in V, t, s$

(7) Since the total number of towers at any location is limited, allocated number of towers at any period cannot exceed that limit.

$\sum_p b_{p,l,t} \leq w_l$ $\qquad\qquad \forall l, t$

4. Experiments and Results

In this section, we present a series of computational experiments designed to evaluate the performance of the proposed solution approach under varying levels of demand uncertainty. Specifically, we investigate how different numbers of scenarios affect the robustness and efficiency of the model's solutions. To this end, we construct a detailed experimental problem instance that simultaneously addresses inventory flow, pricing decisions, and tower allocation. Demand uncertainty is modeled through a scenario-based approach, allowing us to assess the model's ability to generate reliable and high-quality solutions across a range of potential future outcomes.

4.1 Experiment Settings

The experimental problem instance is defined over a single day divided into 24 periods and includes 10 locations—comprising 2 supply centers and 8 vending machine sites. The system manages 8 distinct items, each offered at 3 different price points, resulting in a rich decision space. A total of 28 transportation routes connect the locations, and customer demand is represented through 513 individual demand entries across locations, items, prices, and periods. The average travel cost is assumed to be $1 per kilometer, providing a practical basis for evaluating transportation-related decisions within the model.

Table 1. Parameter values used in the experiments

Item ID	Procurement Cost ($)	Holding Cost ($)	Tower Capacity (unit)	Price 1 ($)	Price 2 ($)	Price 3 ($)
1	1500	15	30	1750	2000	2300
2	1750	17	20	2000	2400	2700
3	500	5	20	700	850	1000
4	200	2	40	250	300	400
5	160	2	50	200	250	350
6	40	1	50	60	80	100
7	400	4	40	500	600	800
8	275	3	80	350	400	500
1	1500	15	30	1750	2000	2300
2	1750	17	20	2000	2400	2700

Table 1 presents key parameters for each item, including procurement cost, holding cost, tower capacity, and three distinct pricing levels. These values define the economic and physical characteristics of the inventory and play a critical role in the optimization model's decisions. The procurement costs vary significantly across items, from as low as $40 to as high as $1,750. Similarly, holding costs are proportional to the value and perishability of items, influencing inventory turnover and replenishment strategies. Tower capacity values indicate how many units of each item can be stored per tower. These vary based on item size and packaging requirements. Tower capacity values indicate how many units of each item can be stored per tower. These vary based on item size and packaging requirements. The three-tiered pricing structure for each item introduces flexibility for dynamic pricing decisions. The range between the lowest and highest prices is particularly notable for high-cost items (e.g., item 2 ranges from $2,000 to $2,700).

Table 2. Information About Locations for the Generated Instance

Location	Number of Towers
SC (1)	0
SC (2)	0
A	5
B	8
C	6
D	5
E	7
F	5
G	4
H	4

Table 2 provides detailed information about the transportation routes in the experimental instance, including the origin and destination of each route, the distance between locations, the number of required periods for transportation, and the specific arrival periods. This information plays a critical role in both cost and timing considerations within the model. Distances directly influence the transportation cost component of the objective function, while arrival periods impact inventory management decisions due to holding costs incurred before items are sold. The presence of multiple arrival periods for a single route introduces a layer of complexity, reflecting real-world logistics constraints such as delivery scheduling, traffic conditions, or resource availability. For example, while the route from SC(1) to F is only 20 km, its arrival can occur at periods 1, 7, or 14, offering flexibility but also requiring strategic planning to balance early delivery (with higher holding costs) against delayed delivery (which might cause stockouts). Similarly, longer routes such as from G to C (70 km) or C to SC(2) (80 km) have longer lead times and fewer arrival options, thus increasing the importance of precise scheduling. Figure 1 illustrates the routes between these locations mentioned in Table 3.

Table 3. Information About Routes for Generated Instance

Origin	Destination	Distance (km)	Required Periods	Arrival 1	Arrival 2	Arrival 3
SC (1)	F	20	0	1	7	14
	H	30	0	2	10	
F	D	10	0	4	16	
D	H	10	0	6	20	
SC (2)	B	10	0	2	13	18
	A	20	0	5	17	
	C	80	1	3	11	
A	E	10	0	7	18	
SC (2)	G	10	0	8	15	
F	SC (1)	20	0	11		
H		30	0	19		
B	SC (2)	10	0	15		
E		30	0	20		
G	C	70	1	9	21	
C	SC (2)	80	1	5	16	

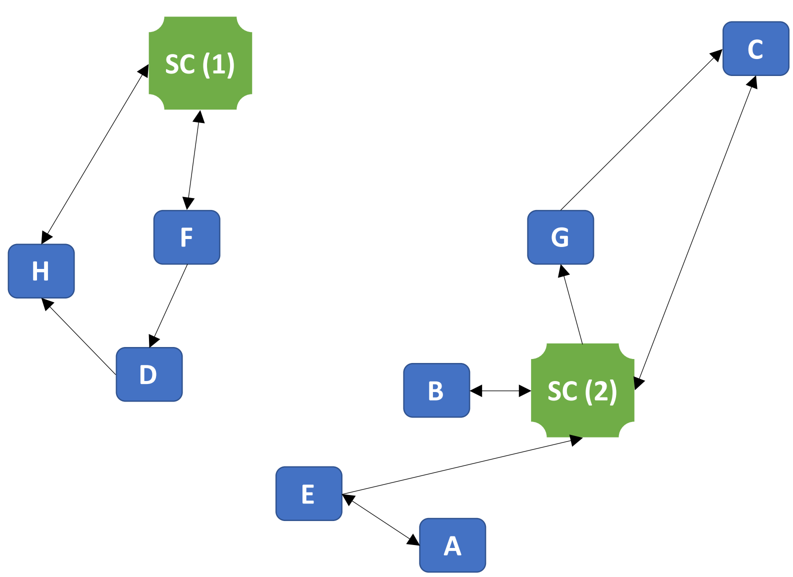

Figure 1. Route Map

Table 4 presents the initial inventory levels of various items at each vending machine location at the beginning of the planning horizon. These initial stock levels form the baseline from which inventory dynamics evolve across the planning periods and scenarios. Since it is assumed that there is no lead time for procurement, supply centers have no inventory. The inclusion of this table is crucial for two reasons: it directly affects early-period sales opportunities and it influences subsequent replenishment and transfer decisions.

The table reveals that the distribution of initial inventories is not uniform across items or locations. This reflects operational constraints such as limited delivery capacity, location-specific demand expectations, and historical consumption patterns. For instance, Location B has initial stock for three items (1, 6, and 8), while other locations like F and G also hold a diverse set of items. In contrast, some items are stocked in only a few locations initially, which may lead to early imbalances in supply and demand, prompting the need for inter-location transfers.

These initial inventories also interact with the model's constraints on tower capacity and replenishment timing. High initial stock can reduce the immediate need for deliveries but may increase holding costs if not sold quickly. Conversely, locations with low or zero initial inventory must rely on timely replenishments or transfers, particularly when customer demand is high.

Table 4. Initial Inventory Level of Locations

Location	Item	Inventory Level
B	1	10
B	6	30
B	8	50
C	1	5
C	6	45
C	7	20
C	5	30
E	6	20
E	2	8
F	8	10
F	5	14
G	6	15
G	8	10
G	7	7

Tables 5, 6, and 7 provide a quantitative overview of the problem size by reporting the number of parameters, decision variables, and constraints in the model.

In particular, Table 5 enumerates the model's parameters, which include demand forecasts, product-specific cost values, capacity constraints, and price levels. The large number of demand values (513) reflects the model's ability to accommodate a rich and detailed representation of customer behavior across products, locations, periods, and price levels.

Table 6 shows the number of decision variables involved in the model. These variables capture key operational decisions such as item pricing, tower allocation, inventory levels, transfer flows, and sales quantities. The presence of thousands of binary and continuous variables (e.g., over 4,600 variables related to pricing and sales) indicates the high dimensionality of the problem. As the number of scenarios increases, so does the number of scenario-dependent variables.

Table 7 reports the number of constraints used to enforce logical and operational feasibility in the model. These include inventory balance, transfer feasibility, sales limits, tower capacity, and pricing exclusivity constraints. The most numerous constraints are related to inventory balance and sales feasibility, which are repeated across products, locations, periods, and scenarios. The number of constraints scales with both the number of items and the number of scenarios, making the formulation computationally intensive, particularly for large-scale instances.

Table 5. Numbers of Parameters

Parameter	Number
$d_{p,r_p,l,t,s}$	513*$\lvert S\rvert$
c_p	8
h_p	8
$c_{l,l}^t$	15
$l_{l,l}^t$	15
w_t	8
y_p	8

Table 6. Numbers of Decision Variables

Decision Variable	Number
$a_{p,r_p,l,t}$	4.608
$b_{p,l,t}$	1.536
$n_{p,l,t,s}$	$1.536 \times \lvert S\rvert$
$r_{p,l,l,t,s}$	$10.752 \times \lvert S\rvert$
$z_{l,l,t,s}$	$1.344 \times \lvert S\rvert$
$u_{p,r_p,l,t,s}$	$4.608 \times \lvert S\rvert$

Table 7. Numbers of Constraints

Constraint	Number
(1)	1.536
(2)	$1.536 \times \lvert S\rvert$
(3)	$2.160 \times \lvert S\rvert - 28$
(4)	$2.160 \times \lvert S\rvert$
(5)	$6.144 \times \lvert S\rvert$
(6)	$3.072 \times \lvert S\rvert$
(7)	192

4.2 Numerical Results

We ran our model for instances with 1, 10, 50, and 100 scenarios, performing each run 10 times. Figures 2, 3, 4, and 5 display the objective function values and run times for these instances, respectively. In the 1-scenario and 10-scenario runs, we observed significant deviations in revenue. However, the model provided more robust results with 50 and 100 scenarios, with run times ranging from 300 to 400 seconds for the 50-scenario case and 1,250 to 3,000 seconds for the 100-scenario case. Therefore, the 50-scenario case appears to be the optimal choice, as its results are as robust as those from the 100-scenario case, but with a significantly lower run time.

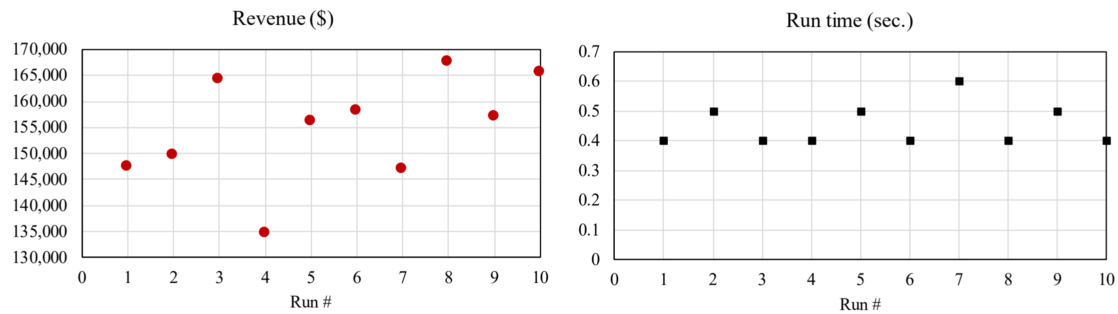

Figure 2. Profit and Run-time for 1 Scenario Case

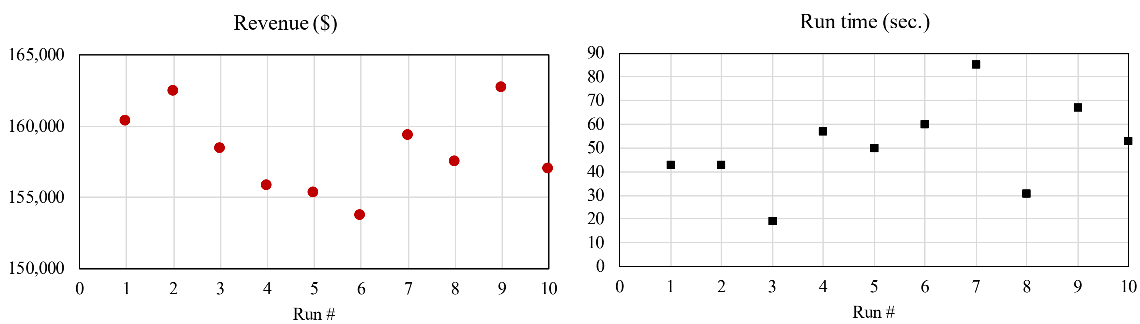

Figure 3. Revenue and Run-time for 10 Scenario Case

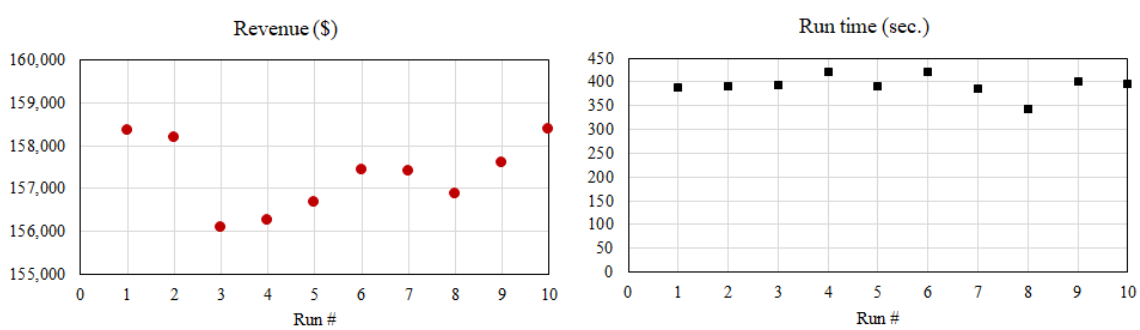

Figure 4. Revenue and Run-time for 50 Scenario Case

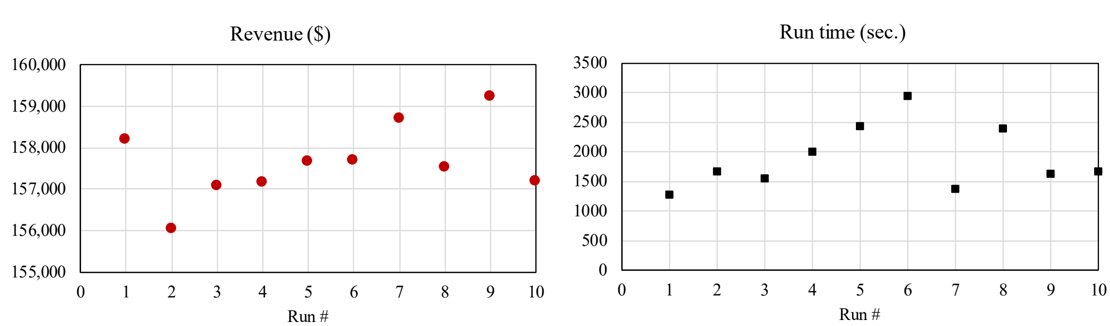

Figure 5. Revenue and Run-time for 100 Scenario Case

Figure 6 compares the revenue and run-time performance of solutions generated using 1, 10, 50, and 100 scenarios. The left plot shows that while the 1-scenario case occasionally produces the highest revenue, its performance varies greatly and drops significantly in several instances, indicating a lack of robustness. In contrast, the 100-scenario case provides the most consistent revenue across all instances, though slightly lower at the high end. The 10- and 50-scenario cases offer a good balance, with more stable revenue than the 1-scenario case and less computational burden than the 100-scenario case. The right plot highlights that run-time increases significantly with the number of scenarios, particularly for the 100-scenario case, which is much more computationally intensive.

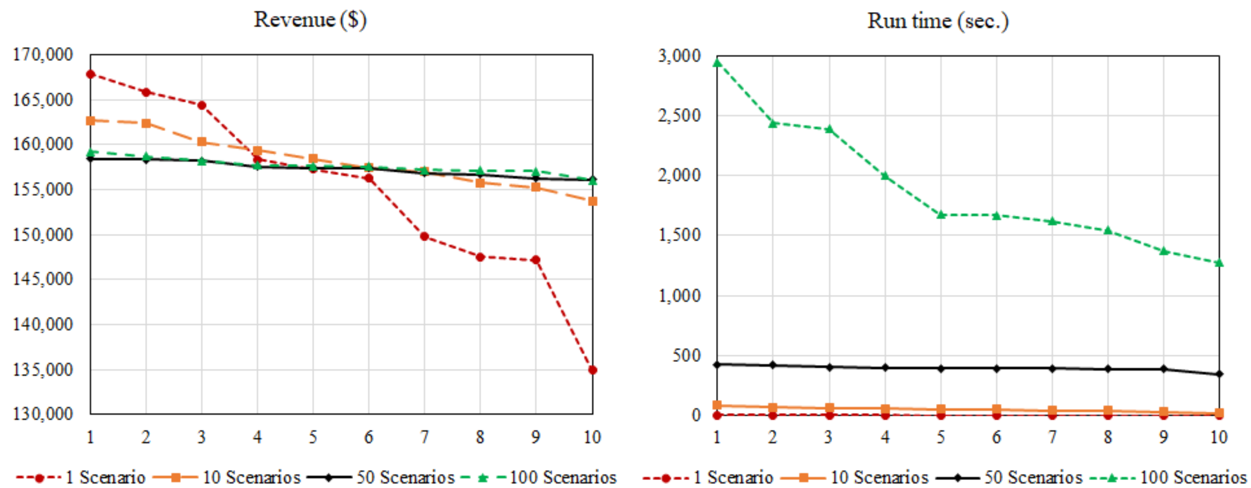

Figure 6. Comparison of revenue and run-time for cases with 1, 10, 50, and 100 scenarios (values are ordered from largest to smallest).

5. Conclusion

In this study, we developed a mathematical model to determine the optimal operational policy for managing multi-item vending machines under stochastic demand. Unlike prior studies that often address pricing, inventory, or routing decisions in isolation, our model integrates these key operational elements into a unified scenario-based framework. It accounts for dynamic pricing, inventory control, space allocation, and inter-location transfers, offering a realistic and holistic approach to vending machine management.

Our experimental results demonstrate that the proposed model delivers robust and efficient solutions, particularly when a sufficient number of demand scenarios are incorporated. Our analysis revealed that using around 50 scenarios strikes a desirable balance between solution robustness and computational efficiency, achieving consistent performance without incurring excessive run times.

This work contributes to the literature by introducing a formulation tailored to real-world vending machine operations. It also provides practical insights for decision-makers seeking to enhance profitability while managing operational complexity in uncertain environments. As future work, a heuristic or meta-heuristic approach could be developed to handle larger instances of the problem. Additionally, vehicle routing could be integrated with the processes already embedded in the model to optimize logistics. Another potential direction for future research could involve experimenting with alternative well-behaved random variables to model the stochastic demand over time, which may provide a more accurate reflection of real-world variability and improve the robustness of the model.

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Biographies

Hacı Şahin received his bachelor's degree from the Middle East Technical University (METU) Department of Industrial Engineering in 2009. He continued his academic studies in the same department and completed his master's degree in 2020. Conducting research in various subfields of industrial engineering, Şahin has academic and professional experience, particularly in operations research, production systems, and decision support systems. He has participated in various projects by effectively utilizing analytical approaches and engineering-based problem-solving techniques. Currently serving as a manager at an electricity distribution company, Hacı Şahin specializes in integrating engineering disciplines into business processes and continues his work on efficiency and operational improvements in the industry.

Serhan Duran holds two masters degrees and a Ph.D. from H. Milton Stewart School of Industrial and Systems Engineering at the Georgia Institute of Technology. During his PhD studies he worked as an operations research analyst on logistics projects for both profit and non-profit organizations. Upon receiving his Ph.D. degree in 2007, Dr. Duran started as a member of the faculty in METU Industrial Engineering Department. His research interest includes revenue management, humanitarian logistics, inventory theory.

Ertan Yakıcı graduated from the Industrial Engineering Undergraduate Program at the Turkish Naval Academy in 1999, the Industrial Engineering Master's Program at the Georgia Institute of Technology, USA, in 2004, the MBA Program at Çankaya University in 2006, and the Industrial Engineering Ph.D. Program at Middle East Technical University in 2013. In 2015-2016, he conducted postdoctoral research at the Naval Postgraduate School, USA. Between 2014 and 2023, he served as a faculty member in the National Defense University, Turkish Naval Academy. In 2024, he joined the Department of Industrial Engineering at Çankaya University as an Associate Professor, where he continues to serve. His primary research focuses on the application of operations research techniques and optimization. He specializes in the areas of facility location and vehicle/inventory routing. His recent research includes UAV mission planning.

Zahra Zare holds a bachelor's and master's degree in Industrial Engineering and is pursuing a Ph.D. in Industrial and Human Factors Engineering from Wright State University. She is also pursuing a master's degree in Marketing Analytics and Insights at Wright State University. She possesses over 10 years of marketing and data analysis experience in the industry. Her research interests include optimization modeling, supply chain analytics, and data-driven decision-making. She has published and co-authored several papers on topics ranging from outsourcing reliability to project scheduling, and her current work includes integrated optimization approaches to urban air mobility systems.

Mumtaz Karatas holds a BSc in Industrial Engineering from the Turkish Naval Academy and an MSc in Industrial & Operations Engineering from the University of Michigan. He has six years of experience as an operations research analyst for the navy and obtained his PhD in Industrial Engineering from Kocaeli University. Following his time as a researcher at the Naval Postgraduate School for two years, he served as a faculty member at the Turkish Naval Academy for 10 years. His primary research focuses on the application of operations research techniques, optimization, machine learning, and data analytics to tackle supply chain design and logistics problems, as well as defense planning problems. He specializes in areas such as facility location and sizing, vehicle/inventory routing, sensor network design, and transportation planning. His recent research includes robust optimization in the context of disaster planning, healthcare resource management, and UAV mission planning.