

# **Quantifying Worst-Case Supply Chain Carbon Footprints: A Network-Based Approach**

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## **Abstract**

This paper explores the worst-case carbon footprint of supply chain networks under uncertain link failure scenarios, with the objective of quantifying the environmental impact in extreme conditions. Supply chain networks are vulnerable to disruptions, such as link failures, which can significantly impact their carbon emissions. These failures, arising from factors like natural disasters, geopolitical risks, or logistical challenges, introduce substantial uncertainty in the network's performance. To capture the environmental impact under extreme failure conditions, we adopt a network-based approach, leveraging robust optimization to handle the uncertainty and mixed integer non-linear optimization (MINLP) to solve the resulting complex problem. The study models the supply chain as a network of nodes and edges, where nodes represent supply chain entities (e.g., suppliers, warehouses, and consumers) and edges represent transportation or logistics links. The uncertainty in the failure of these links is captured through a set of possible failure scenarios, which may grow exponentially in size. Robust optimization techniques are employed to determine the worst-case carbon footprint, considering the full range of failure scenarios, while ensuring the solution remains feasible under all possible conditions. MINLP is used to solve the optimization problem, identifying solutions that minimize carbon emissions while addressing the uncertainty in the network. The results provide key insights into managing supply chain emissions in the presence of large uncertainty, helping supply chain managers design more resilient and sustainable networks that can adapt to extreme disruptions while minimizing their environmental impact.

## **Keywords**

Worst-case carbon footprint, Supply chain modeling and optimization, Robust optimization, Sustainable supply chain networks, Mixed integer nonlinear optimization.

## **1. Introduction**

Supply chains are the backbone of global commerce, enabling the movement of goods and services across vast networks that connect suppliers, manufacturers, warehouses, and retailers. However, these networks are increasingly vulnerable to disruptions such as natural disasters, geopolitical risks, and logistical challenges (Christopher 2016). These disruptions often manifest as link failures within the supply chain, which can lead to significant operational and environmental consequences. A particularly critical concern is the impact of such disruptions on the carbon footprint of supply chain networks. Transportation and logistics activities, which constitute a substantial portion of global greenhouse gas emissions, are highly susceptible to such failures (ManMohan S. Sodhi 2012). Understanding the worst-case carbon footprint under uncertain link failure scenarios is vital for designing supply chains that are both resilient and environmentally sustainable (Ivanov 2020).

Traditional methods of supply chain modeling often assume deterministic conditions, which do not adequately capture the risks posed by uncertainties in the network, particularly those related to disruptions (Sawik 2017). In reality, the uncertainty set of possible failure scenarios in large-scale supply chains can grow exponentially, presenting a significant challenge for optimization (see for instance (Chang 2019, Chandra 2022)). As a result, it is essential to incorporate robust optimization techniques that can account for this uncertainty while minimizing environmental impact (Ben-Tal 2010). This paper introduces a network-based approach that combines robust optimization with mixed integer non-linear optimization (MINLP) to model the worst-case carbon footprint of supply chains under uncertainty.

In our approach, the supply chain is modeled as a network of nodes and edges, where nodes represent supply chain entities such as suppliers, warehouses, and consumers, and edges represent the transportation or logistical links between them. Uncertainty in the failure of these links is modeled as a set of possible failure scenarios, with an exponentially growing size due to the complexity of modern supply chains (A. Chandra 2024). To address this, robust optimization is employed to determine the worst-case carbon footprint across all failure scenarios, ensuring that the network's carbon emissions are minimized even under the most extreme conditions (Bertsimas 2004). Mixed integer non-linear programming (MINLP) is then applied to solve the resulting complex optimization problem, identifying the most efficient configurations that balance emissions reduction with the resilience of the network. The results of this study contribute valuable insights to the field of sustainable supply chain management, providing actionable strategies for minimizing carbon emissions in the face of uncertainty. Our approach equips supply chain managers with the tools to design networks that are not only resilient to disruptions but also aligned with sustainability goals in an increasingly unpredictable global environment.

Supply chain networks play a crucial role in global trade and logistics, yet they are also significant contributors to carbon emissions. Transportation, warehousing, and manufacturing activities within supply chains account for a substantial portion of global greenhouse gas (GHG) emissions (McKinnon 2018, Crainic 2023). Given the increasing urgency to decarbonize supply chains, it is essential to assess and mitigate carbon footprint under worst-case operational conditions, particularly in the presence of uncertainty and disruptions. However, traditional optimization approaches for supply chain design often fail to incorporate uncertainties in network failures, leading to underestimated emissions and ineffective carbon management strategies when disruptions occur.

A widely used method for analyzing network resilience and emissions impact is exhaustive enumeration, wherein all possible failure scenarios are explicitly listed, and the carbon footprint associated with routing decisions under each failure case is computed (Matisziw 2009, Albrecht 2015, Ben Hammouda 2020). This approach has been applied in supply chain network design and transportation planning, where researchers evaluate network robustness by determining emissions under different disruption scenarios (Demir 2014, Guo 2023). However, exhaustive enumeration is computationally prohibitive for large-scale networks due to the exponential growth in failure scenarios. The number of scenarios to be evaluated increases rapidly as the number of supply chain links grows, making it infeasible for real-world supply chains with extensive global operations. For instance, in a supply chain network with  $|E|$  transportation links, evaluating the carbon footprint under  $b$ -simultaneous link failures requires considering  $\binom{|E|}{b}$  failure scenarios. As the network expands, the number of failure scenarios explodes exponentially. To see this, consider the following service network in Figure 1, with six edges and four nodes (A, B, s, d). There is one source (s) - destination (d) node pair that has a demand requirement to be met. Assuming that the network is subjected to all  $b = 1$  simultaneous link failure scenarios, we wish to compute the worst-case carbon footprint under single-link failures, ensuring that the demand requirements are met. With the exhaustive enumeration approach, we first list out all  $\binom{6}{1}$  failure scenarios in Figure 2. We note that in Figure 2, there are six graphs each corresponding to one of the six failure scenarios. The red colored link in each of the graphs denote the non-functional link in that failure scenario. Once the enumerated list of all single link failure scenarios is complete, we then compute the carbon footprint for each of the six failure scenarios. The worst-case emissions analysis proceeds by computing the carbon footprint for the optimal routing decision in each failure scenario and identifying the scenario that results in the highest emissions.

In a realistic global supply chain network with 50 transportation links, assessing two-simultaneous link failures would require evaluating  $\binom{50}{2} = 19600$  failure scenarios. As noted in prior studies on green logistics and sustainable supply chains, such large-scale scenario enumeration is infeasible due to its overwhelming computational complexity (Snyder

2016, Hosseini 2016). Furthermore, exhaustive enumeration may fail to identify true worst-case emissions scenarios, as it does not prioritize the most carbon-intensive failure events but instead treats all disruptions equally (T. G. Crainic 2007).

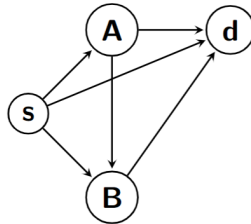


Figure 1: Example of a service network

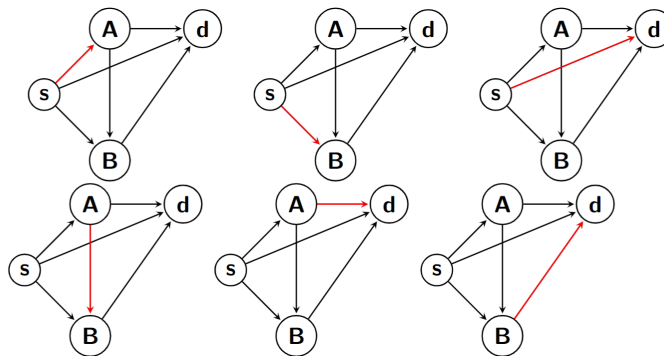


Figure 2: All one link failure scenarios (Red: Failed Link)

### 1.1 Minimizing Carbon footprint: An optimization approach

Disruptions in supply chain networks directly impact carbon emissions by forcing the rerouting of goods along longer or less energy-efficient paths, increasing fuel consumption and GHG emissions (E. T. Demir 2014, Quariguasi Frota Neto 2010). For example, when a primary low-emission transport route (such as rail) becomes unavailable due to infrastructure failure, freight may need to be rerouted via high-emission alternatives, such as trucking or air freight. Similarly, warehouse failures may force companies to consolidate shipments at alternate hubs, leading to increased distances and higher emissions. Studies in resilient supply chain design have shown that failure-induced rerouting can increase emissions by up to 30% in some cases (Jabbarzadeh 2018). This issue is particularly critical given the recent emphasis on corporate sustainability commitments and carbon pricing policies. Many companies are now required to report their Scope 3 emissions, which include transportation-related emissions across their supply chain (CDP, 2021). Under extreme disruption scenarios, failure to account for worst-case emissions can result in regulatory non-compliance and financial penalties. Moreover, carbon taxation mechanisms, such as the EU Emissions Trading System (ETS), impose additional costs on high-emission supply chains, further emphasizing the need for robust emissions-aware routing (Zhang 2022).

Given the limitations of exhaustive enumeration, an optimization-driven methodology is needed to efficiently quantify and minimize the worst-case carbon footprint in supply chain networks under uncertainty. Instead of evaluating all possible failures, robust optimization offers a more scalable alternative by modeling uncertainty through worst-case failure sets and optimizing routing accordingly (D. D. Bertsimas 2011). Additionally, mixed-integer non-linear programming (MINLP) techniques provide a more accurate representation of carbon emissions functions, incorporating non-linear fuel consumption dynamics, congestion effects, and energy-efficiency trade-offs (Bektaş 2011). This paper proposes a network-based robust optimization framework that determines the worst-case carbon footprint while ensuring that supply chain operations remain resilient and environmentally sustainable. Our approach builds upon advancements in green supply chain modeling (Tang 2012), resilient logistics (Nagurney 2012), and

stochastic optimization for emissions reduction (Shapiro 2021) to develop a scalable solution to this critical problem. By integrating robust optimization and MINLP, we aim to capture a broad uncertainty set of failure scenarios without relying on exhaustive enumeration, allowing for efficient identification of worst-case emissions scenarios and optimal routing decisions. In the following section, we introduce our mathematical formulation and solution methodology for worst-case carbon footprint quantification in supply chain networks

## **2. Robust optimization for quantifying worst-case**

Robust optimization (RO) has emerged as an effective framework for handling uncertainty in supply chains (D. D. Bertsimas 2011, A. A. Ben-Tal 2009). Traditional supply chain optimization approaches often rely on deterministic models that assume full availability of network links and predictable demand. However, real-world supply chains are subject to unforeseen disruptions, which can significantly impact logistics, increase operational costs, and, most critically, lead to higher carbon emissions due to longer and less efficient rerouting (Snyder 2016). Unlike stochastic optimization, which requires detailed probabilistic information about failures, RO operates under a worst-case scenario framework, ensuring that supply chain decisions remain feasible and cost-effective even under extreme disruption conditions. Stochastic models, while useful, depend on accurate probability distributions of failures, which may be difficult to estimate and highly uncertain in practice (Sahinidis 2004). In contrast, RO does not require explicit probability distributions; instead, it defines an uncertainty set of possible disruptions and optimizes the decision-making process to minimize the impact of the worst-case scenario. This makes RO particularly suitable for quantifying and mitigating the worst-case carbon footprint of supply chain networks, where link failures could lead to exponential increases in emissions due to rerouting through longer paths or reliance on higher-emission transport (Gendreau 2016). In the context of supply chain sustainability, robust optimization has been widely adopted to model uncertainty in various applications, such as facility location problems (Gopalakrishnan 2010) inventory control (D. a. Bertsimas 2006), and network design. However, its application to carbon footprint minimization under large-scale link failures has received relatively less attention. This study extends the existing body of work by incorporating a robust framework to quantify the worst-case carbon footprint of supply chain networks under exponentially growing failure scenarios. Given that the number of possible disruptions increases combinatorially with the number of links in the network, standard enumeration techniques become computationally intractable (Matisziw 2009). By leveraging robust optimization, we develop an approach that efficiently identifies the most emission-efficient routing strategies, ensuring supply chain resilience and sustainability in the face of extreme disruptions.

This study contributes to the literature in multiple ways:

- 1) Quantification of worst-case carbon emissions in disrupted supply chain networks, providing actionable insights for sustainability.
- 2) Development of a scalable robust optimization model to handle exponential failure scenarios efficiently.
- 3) Integration of mixed-integer nonlinear programming (MINLP) techniques to solve complex routing problems with emissions constraints.
- 4) Bridging the gap between network performance with respect to environmental sustainability.

By incorporating robust optimization into supply chain network design, firms can proactively anticipate high-carbon footprint scenarios and mitigate their impact, aligning operational resilience with environmental goals. Future research directions could further explore adaptive robust optimization techniques that dynamically adjust supply chain decisions in response to real-time disruptions and evolving carbon policies. The general structure of a robust optimization problem is obtained as follows.

$$R^* = \max_{x \in X} \min_{y \in Y(x)} f(x, y)$$

The above problem is a two-stage optimization problem, where the outer maximization happens over the variables  $x \in X$ . Moreover, for any given  $x' \in X$ , the inner minimization aims to minimize  $f(x', y)$  over the variables  $y \in Y(x')$ . Typically, as is the case with our problem as well, for a given  $x \in X$ , the inner optimization problem is a linear program (LP) in variables  $y$ .

## **3. Modeling Worst-case Carbon footprint**

We consider standard supply chain networks with four levels: suppliers, plants, warehouses, and retailers. These are also commonly referred to as four-echelon supply chain networks (Khalifehzadeh 2015, Pan 2013, Wenbing 2013).

We assume that the demands only occur at the retailers and that there are no fixed costs at any of the facilities. These assumptions are standard in the literature, see for reference (Adenso-Díaz 2018, A. a. Chandra 2024). We denote a given SCN as a graph  $G(V, E)$ , where  $V$  is the set of nodes, consisting of supplier nodes ( $S$ ), plant nodes ( $P$ ), warehouse nodes ( $W$ ), and retailer nodes ( $R$ ) such that  $V = S \cup P \cup W \cup R$ . The set of edges is represented by  $E$ , such that  $E = E_{SP} \cup E_{PW} \cup E_{WR}$ , where  $E_{SP}$  represents the set of directed edges connecting the suppliers to the plants,  $E_{PW}$  is the set of directed edges connecting the plants to the warehouses, and  $E_{WR}$  represents the set of directed edges connecting the warehouses and the retailers (see for instance Figure 3). Figure 3 shows a supply chain network, with suppliers =  $\{s1, s2, s3, s4\}$ , plants =  $\{p1, p2, p3\}$ , warehouses =  $\{w1, w2\}$ , and retailers =  $\{w1, w2, w3, w4\}$ . The directed edges represent the flow of material in the supply chain network.

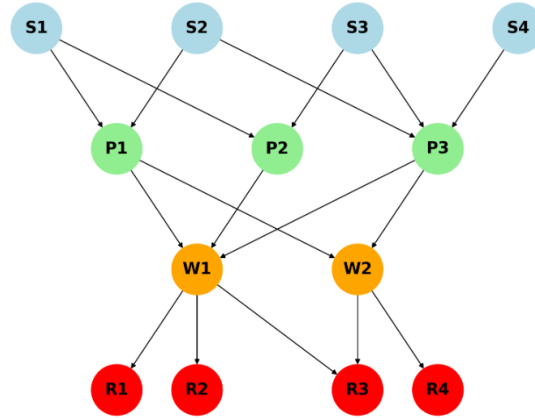


Figure 3: Four Echelon Supply Chain Network

The state of a link  $(i, j)$  under the failure scenario  $x$  is given by  $x_{ij} \in \{0,1\}$  where  $x_{ij} = 1$  indicates that the link  $(i, j)$  has failed and  $x_{ij} = 0$  indicates that  $(i, j)$  is alive under scenario  $x$ . Given a failure scenario  $x$ , the network can reroute traffic. The most flexible response is to solve a multicommodity flow problem for the scenario  $x$  (Chang, et al. 2019). We leverage this formulation to model the network routing mechanism, which minimizes the total carbon emission for the network. Table 1 and Table 2 respectively provide the list of parameters and variables used in the formulation.

Table 1: List of notations

Notation	Meaning of the notation
S, P, W, R	Known set of supplier, plant, warehouse, and retailer nodes
V	Set of all nodes in the SCN. $V = S \cup P \cup W \cup R$
s	Supplier s (s - Index for suppliers)
p	Plant p (p - Index for plants)
w	Warehouse w (w - Index for warehouses)
r	Retailer r (r - Index for retailers)
$E_{SP}$	Known set of edges from supplier nodes (S) to plant nodes (P)
$E_{PW}$	Known set of edges from plant nodes (P) to warehouse nodes (W)
$E_{WR}$	Known set of edges from warehouse nodes (W) to retailer nodes (R)
E	Set of all edges in the SCN. $E = E_{SP} \cup E_{PW} \cup E_{WR}$
$c_{ij}$	Known maximum flow capacity of the link $(i,j) \in E$
$D_r$	Known demand requirement of retailer r
$O_{ij}$	Carbon emission coefficient (Kg CO2 per unit routed on link $(i,j)$ )

Table 2: List of variables

Variable	Definition of the variable
$y_{ij}$	Flow routed on link $(i,j)$
x	Link failure scenario capturing the state of all the links in the network

Given a set of failure scenarios X, our objective is to compute the worst-case carbon footprint emission in satisfying the demand requirements among all failure scenarios  $x \in X$ . The set X, is referred to as the set of uncertain failures. For our work we will be focusing on the set where  $f_1$  links among  $E_{SP}$ ,  $f_2$  links among  $E_{PW}$ , and  $f_3$  links among  $E_{WR}$  fail simultaneously. This is a commonly used uncertainty set popular across the supply chain networking and telecommunication community, see (A. Chandra 2024, Wang 2010, Adenso-Díaz 2018, Poudel 2016) for reference.

$$(M:) \min_y \sum_{(i,j) \in E} y_{ij} O_{ij} \quad (1)$$

$$y_{ij} \leq (1 - x_{ij}) c_{ij} \quad (2)$$

$$\sum_{w \in W: (w,r) \in E} y_{wr} = D_r \quad \forall r \in R \quad (3)$$

$$\sum_{s \in S: (s,p) \in E} y_{sp} = \sum_{w \in W: (p,w) \in E} y_{pw} \quad \forall p \in P \quad (4)$$

$$\sum_{p \in P: (p,w) \in E} y_{pw} = \sum_{r \in R: (w,r) \in E} y_{wr} \quad \forall w \in W \quad (5)$$

$$y_{ij} \geq 0 \quad \forall (i,j) \in E \quad (6)$$

The uncertainty set where  $f_1$  links among  $E_{SP}$ ,  $f_2$  links among  $E_{PW}$ , and  $f_3$  links among  $E_{WR}$  fail simultaneously is defined as follows:

$$X = \{x_{ij} \in \{0,1\} \forall (i,j) \in E: \sum_{(i,j) \in E_{SP}} x_{ij} = f_1, \sum_{(i,j) \in E_{PW}} x_{ij} = f_2, \sum_{(i,j) \in E_{WR}} x_{ij} = f_3\} \quad (7)$$

So eventually we are interested to solve the following two stage optimization problem as defined in (6), where the outer maximization capturing the worst-case network performance in terms of the total cost of routing occurs over  $x \in X$ . The inner minimization, for a given  $x \in X$ , solve the problem defined as in M.

$$\max_x \min_y \sum_{(i,j) \in E} y_{ij} O_{ij} \text{ subject to } \{(2), (3), (4), (5), (6), (7)\} \quad (8)$$

### 3.1 Renormalizing to single-stage optimization problem

We leverage the linear optimization duality techniques to solve the above two stage optimization problem as a single stage optimization problem. For a fixed  $x \in X$ , we dualize the inner minimization problem, and due to the zero-duality gap, we can equivalently write the two-stage optimization problem as a single stage problem described as (M1).

$$(M1:) \quad \max_{\alpha, \gamma, \theta, \lambda, x} \sum_{r \in R} \lambda_r D_r + \sum_{(i,j) \in E} (x_{ij} c_{ij} \alpha_{ij} - c_{ij} \alpha_{ij}) \quad (9)$$

$$\begin{aligned} -\alpha_{sp} + \gamma_p &\leq O_{sp} & \forall (s,p) \in E_{SP} & (10) \\ -\alpha_{pw} - \gamma_p + \theta_w &\leq O_{pw} & \forall (p,w) \in E_{PW} & (11) \\ -\alpha_{wr} - \theta_w + \lambda_r &\leq O_{wr} & \forall (w,r) \in E_{WR} & (12) \\ \alpha_{ij} &\geq 0 & \forall (i,j) \in E & (13) \\ x &\in X \end{aligned}$$

## 4. Evaluation

In this section we provide numerical results for the problem of evaluating the worst-case carbon footprint in supply chain networks admits b-simultaneous link failure scenarios. We solve this problem using the following approaches already discussed in the paper and compare their optimal values and the CPU runtimes.

1. Enumeration approach. (Referred to as approach 1 – A1)
2. Single-stage optimization using M1. (Referred to as approach 2 – A2)

We assume link capacities  $c_{ij}$  for all link  $(i,j) \in E$  to be uniformly chosen in the interval **[40, 50]**. The demand requirement at each retailer  $r$ , represented by  $D_r$  is also drawn uniformly from the interval **[20, 40]**. The carbon footprint associated with transportation of one unit of traffic on each link is represented as  $O_{ij}$ , which we assume follows a uniform distribution over the interval **[1, 2]** Kgs of CO<sub>2</sub>. In Table 3, we list out the testing topologies. In Column labeled “T” we list out the supply chain networks we test our approach on. Under Columns labeled “|S|”, “|P|”, “|W|”, and “|R|” we report the number of source nodes, plant nodes, warehouse nodes, and retailer nodes. The Column labeled “(|E<sub>SP</sub>|, |E<sub>PW</sub>|, |E<sub>WR</sub>|)” we report the cardinality of the edge sets  $E_{SP}$ ,  $E_{PW}$ , and  $E_{WR}$ , where  $|E| = |E_{SP}| + |E_{PW}| + |E_{WR}|$ . We also report the total number of vertices in the supply chain network as  $|V|$ . Assuming that the demand at the retailers are uniformly distributed from the interval **[20, 40]**, under Column T-Dem we report the total demand requirement across all the retailers i.e.,  $T\text{-Dem} = \sum_{r \in R} D_r$ .

Table 3: Supply chain networks for evaluation

T	S	P	W	R	( E <sub>SP</sub>  ,  E <sub>PW</sub>  ,  E <sub>WR</sub>  )	V ,  E	T-Dem
T1	3	4	3	5	(12, 12, 15)	(15, 39)	154.05
T2	5	3	7	3	(15, 21, 21)	(18, 57)	96.57
T3	4	5	8	2	(20, 40, 16)	(19, 76)	62.36
T4	3	8	4	6	(24, 32, 24)	(21, 80)	187.62
T5	8	12	7	6	(96, 84, 42)	(33, 222)	185.85

From the computational standpoint, all the optimization problems solved to get the results in Section 4.1, were first modeled in Python and then solved using Gurobi 10.0 (Gurobi Optimization 2022). The computations were done on a machine with an Intel Xeon E5-2623 CPU @ 3.00 GHz.

#### 4.1 Numerical Results

In this section, we present the evaluation results of our proposed approach, comparing it with the exhaustive enumeration method. Table 4 summarizes the findings, where the first column, labeled “**T** (**f<sub>1</sub>**, **f<sub>2</sub>**, **f<sub>3</sub>**),” details the supply chain network topology and the number of failures occurring in the edge sets **E<sub>SP</sub>**, **E<sub>PW</sub>**, and **E<sub>WR</sub>**, respectively. The column “**A1**” reports the worst-case carbon footprint obtained using the enumeration approach, while “**T-A1**” provides the CPU runtime required for its computation. Similarly, “**A2**” presents the worst-case carbon footprint obtained using our proposed single-stage optimization model (M1), and “**T-A2**” reports its corresponding CPU runtime. The CPU run-times reported in Columns “**T-A1**” and “**T-A2**” are in seconds. The column “**T-Imp%**” quantifies the computational efficiency improvement of our method over the enumeration approach, calculated as:

$$T.Imp\% = \frac{(T.A1 - T.A2)}{T.A1} * 100$$

From the results, we observe that the worst-case carbon footprint values in columns **A1** and **A2** are identical, confirming that our optimization model achieves the optimal solution. However, our method significantly reduces computational effort, as indicated by the consistently lower CPU runtime in **T-A2** compared to **T-A1**. The notable percentage improvements reported in **T-Imp%** demonstrate the effectiveness of our approach in efficiently handling large-scale failure scenarios while ensuring optimality in worst-case carbon footprint quantification. Entries reported as \* in Table 4, indicate the fact that the worst-case carbon footprint for T5 for failures (1, 2, 1) did not get computed in a time limit of 10 hours.

Table 3: Optimal routing cost values via different approaches

<b>T</b> ( <b>f<sub>1</sub></b> , <b>f<sub>2</sub></b> , <b>f<sub>3</sub></b> )	<b>A1</b>	<b>T-A1</b>	<b>A2</b>	<b>T-A2</b>	<b>T-Imp %</b>
T1(1, 1, 1)	<b>603.06</b>	2.59	<b>603.06</b>	0.80	69.11
T1(1, 2, 1)	<b>618.96</b>	14.26	<b>618.96</b>	6.91	51.54
T1(2, 2, 1)	<b>638.44</b>	78.80	<b>638.44</b>	33.30	57.74
T2(0, 2, 1)	<b>343.18</b>	7.07	<b>343.18</b>	3.89	44.98
T2(2, 1, 1)	<b>354.04</b>	71.59	<b>354.04</b>	25.92	63.80
T2(2, 2, 1)	<b>363.31</b>	803.64	<b>363.31</b>	197.90	75.37
T3(0, 0, 1)	<b>216.05</b>	0.07	<b>216.05</b>	0.05	28.57
T3(0, 2, 1)	<b>230.07</b>	23.35	<b>230.07</b>	8.60	63.17
T3(2, 2, 1)	<b>236.21</b>	8168.15	<b>236.21</b>	1177.37	85.59
T4(1, 1, 1)	<b>231.84</b>	89.84	<b>231.84</b>	7.58	91.56
T4(1, 2, 1)	<b>710.76</b>	2410.59	<b>710.76</b>	235.23	90.24
T4(2, 1, 2)	<b>726.11</b>	7487.19	<b>726.11</b>	1029.55	86.25
T5(1, 1, 1)	<b>627.18</b>	1037.50	<b>627.18</b>	346.42	66.61
T5(1, 1, 2)	<b>633.77</b>	15964.74	<b>633.77</b>	2200.36	86.22
T5(1, 2, 1)	*	*	<b>632.30</b>	11990.72	*

#### 5. Conclusion

This study presents a robust optimization framework for quantifying the worst-case carbon footprint of supply chain networks under uncertain link failures. Given that supply chain disruptions can arise from a wide range of factors, such as natural disasters, geopolitical tensions, and logistical breakdowns, understanding the worst-case environmental impact is crucial for designing resilient and sustainable networks. Traditional enumeration-based approaches, while effective in theory, become computationally infeasible due to the exponential growth in the number of failure scenarios. To overcome this challenge, we developed a single-stage optimization model leveraging robust optimization principles to efficiently determine the worst-case carbon footprint without requiring exhaustive scenario enumeration. Our numerical results demonstrate that the proposed approach achieves the same optimal worst-case carbon footprint as enumeration-based methods but with significantly lower computational cost. By comparing CPU

runtime across different network topologies and failure scenarios, we observed substantial improvements in computational efficiency, highlighting the scalability of our method for large supply chain networks. These findings underscore the potential of robust optimization in mitigating the environmental impact of supply chain disruptions while ensuring operational feasibility under worst-case conditions. Future research directions include extending the model to incorporate dynamic failure scenarios, multi-modal transportation networks, and additional sustainability constraints such as carbon pricing mechanisms. Additionally, integrating machine learning techniques to predict high-risk failure scenarios could further enhance decision-making efficiency. Overall, this study provides a foundational step toward developing resilient and low-carbon supply chain networks in an era of increasing uncertainty.

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## **Biography**

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