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Evaluation of Multistage Interconnection Networks Reliability using Monte Carlo Method

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Abstract

Multistage Interconnection Network (MIN) consists of layers of switching elements connected together in a predefined topology providing the connectivity between input and output stages. Reliability evaluation of MIN is important as it determines the usability and efficiency of the network to provide services. This paper presents a method of estimating the reliability of MIN using Monte Carlo method with stratified sampling. Confidence interval of the point estimate is then derived using non-parametric bootstrapping.

Keywords
Multistage Interconnection Network, Monte Carlo, reliability, shuffle exchange network, switching element.

1. Introduction
Multistage Interconnection Network (MIN) falls within the category of indirect network (Prakash et.al, 2020). It has been used in both circuit switching and packet switching networks with the introduction of buffered switches (Rajkumar, 2016). These include multiprocessor and communication network environments such as Ultracomputer (Gottlied, 1987) NEC Cenju-3, Cenju-4 (NEC Corporation, 2003), IBM RP3, ATM switches (Sibal and Zhang, 1995), Gigabit Ethernet (Yu, 1998) and optical network (Yang, 2000). The number of stages, interconnection design and the type of switching element (SE) used in the network configuration differentiate each MIN, for example shuffle exchange network, gamma network (Vama and Raghavendra, 1985), extra stage gamma network (Lee and Hegazy, 1988), delta network, Tandem-Banyan network (Sibal and Zhang, 1995) and multilayer MIN (Tutsch and Gunter, 2003).

The variety and the extensive usage MIN prompt for a method that could provide efficient evaluation of various MIN reliabilities in order to select the best MIN topology (Jahanshahi and Bistouni, 2014). Various methods have been used to evaluate the reliability of a network such as neural network (Srivaree-ratana and Smith, 2002), derivation of bounds (Konak and Smith, 2003; Gunawan, 2002) and sum of disjoint product (Chua and Kuo, 1994). This paper presents a method to estimate the reliability of MIN using Monte Carlo method. A single type of MIN, known as shuffle exchange network with an additional stage (SEN+) that is specifically for multiprocessor environment is discussed. The layout of the MIN topology is shown in Figure 1 with number of inputs, N = 8. The rectangles in the figure represent the 2x2 SEs which provide the interconnection between inputs and outputs.

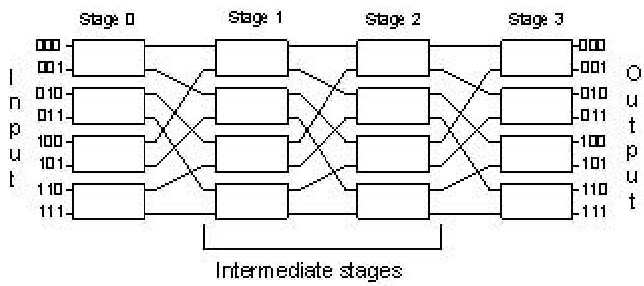

Figure 1. 8x8 SEN+ topology.

A working SE can be in any of the four connection patterns as shown in Figure 2.

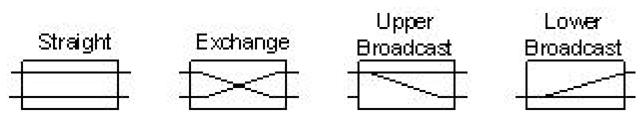

Figure 2. Connection pattern for 2x2 SE.

SEN+ is a hybrid of generic shuffle exchange network (SEN) with higher failure tolerance than SEN. It has two disjoint redundant paths in the intermediate stages thus able to tolerate a single path failure.

2. Reliability of Multistage Interconnection Network (MIN)

Evaluation of MIN reliability in this paper includes three types of reliability: terminal, broadcast and network reliability. Terminal reliability of a MIN is defined as the probability of existence of at least one fault free path between a designated pair of input and output terminals (Prakash et.al., 2020). It is usually used to gauge the robustness of a MIN. Broadcast reliability of a MIN represents the probability that a single input terminal is able to broadcast data to all the output terminals in the network. Network reliability of a MIN is defined as the probability that all input terminals are connected to all output terminals.

3. Monte Carlo Method

Exact reliability of SEN+ can be determined by evaluating all possible SE states but it is NP-hard due to the huge possibilities of SE states as the number of inputs increases. Monte Carlo method is able to provide a point estimate of SEN+ reliability without evaluating every possible SE state. It is based on the adaptation of a method proposed by Fishman (Fishman, 1996). Monte Carlo (MC) method enables estimation of SEN+ reliability via random sampling of SE states. The following assumptions are defined to facilitate the estimation of SEN+ reliability:

i. A SE can only have two states; working=1 or fail=0.
ii. All SE failures are statistically independent and random. A SE is assumed failed when it could not be in any of the four connection patterns; lower broadcast, upper broadcast, straight or exchange pattern (Figure 2).
iii. SE is assumed to be less reliable than the link and cannot be repaired.
iv. All SEs have identical reliability.
v. All SEs in the first and last stages are assumed to be working.

Algorithm 1: Monte Carlo Method (MC) for SEN+

Parameters:

1. Number of SEs in the intermediate stages, n_{im}
2. SE reliability, r(t)
3. Number of inputs, N
4. Number of replications, n_r
5. Number of SE in the first and last stages, n_{fl}.
6. Reliability of SEN+, R

Procedure:

1. SET accumulated reliability, $R_{ac} = 0$
   SET number of working switches, $n_{working} = 1$
   SET number of SE in intermediate stages, n_{im}
   SET total number of samplings, $n_{sampled} = 0$
   SET total connected network, $n_{connected} = 0$

2. REPEAT
   Note: Calculate the stratum sampling size for each stratum. Number of stratum depends on the number of working SEs in the intermediate stages.
   SET number of sampling for stratum i (i = $n_{working}$),

   $$n_{stratum\_size} = n_r \cdot \binom{n_{im}}{n_{working}} . r(t)^{n_{im}} . [1- r(t)]^{n_{working}}$$

   SET $n_{sampled} = n_{sampled} + n_{stratum\_size}$

   Note: Evaluate only when the number of working SEs in the intermediate stages is at least half of the total number of the SEs in the intermediate stages. The SEN+ fails when the number of working SEs in the intermediate stages is less than half of its total.
   IF $n_{working} \geq n_{im}$ x 0.5 THEN
   Note: The interconnection still functions even there is a single SE failure. Evaluation is skipped as the interconnection is functioning when there is only a single SE failure.
   IF $n_{working} < n_{im}$ -1 THEN
   Note: Generated SE states are dependent on the type of interconnection; terminal, broadcast or network.
   Randomly generate SE states in intermediate stages in array state[n_{im}]

   Note: Evaluation of SEN+ network is dependent on the type of interconnection; terminal, broadcast or network. This is done by evaluating the array state[n_{im}].
   IF the SEN+ network is connected THEN
   $n_{connected} = n_{connected} + 1$
   END IF
   ELSE
   $n_{connected} = n_{connected} + n_{stratum\_size}$
   END IF
   $n_{working} = n_{working} + 1$
   UNTIL ($n_{working} \leq n_{im}$)
3. *Note*: The estimated reliability for intermediate stages is multiplied with all the SE reliability for first and last stages to calculate the overall estimated reliability.
   RETURN R = ($n_{connected}$ / $n_{sampled}$) . $r(t)^{n_{fl}}$

Algorithm 1 shows the procedure to perform Monte Carlo method with stratified sampling. Stratified sampling allows us to achieve better approximation of the exact SEN+ reliability. It partitions the sample into several stratums, where each stratum contains homogenous elements. This allows sampling to be performed on important stratums and ignores irrelevant ones, thus improving the accuracy and efficiency of the estimation. Stratum sampling size is based on proportional allocation derived from binomial probability distribution which is defined as

$$n_{stratum\_size} = n_r \binom{n_{im}}{n_{working}} r(t)^{n_{im}} [1- r(t)]^{n_{working}}$$

4. Confidence Interval for Monte Carlo Point Estimate

Confidence interval (Upper Limit, UL and Lower Limit, LL) of the point estimate reliability value using Monte Carlo method is derived using statistical non-parametric bootstrapping method (Efron and Tibshirani, 1993; Wang and Rao, 1997). Non-parametric bootstrapping does not require any assumptions being made on the distribution pattern thus removing any errors that may result biased outcome. The bootstrapping method used to estimate the confidence interval in this paper is based on Efron's percentile confidence limit.

5. Numerical Results

We implemented several other methods on single software platform to gauge our level of accuracy by comparing the results. These include exact terminal reliability of SEN+ calculated using the mathematical (Math) approach (Gunawan, 2002; Thanawastien, 1982), Fard and Gunawan's method (Fard and Gunawan, 2001) to calculate exact broadcast and network reliabilities up to N = 16 inputs and Blake and Trivedi's network reliability bounds (Blake and Trivedi, 1989). For higher number of inputs, we compare our results against Cheng and Ibe's results published in their paper (Cheng and Ibe, 1992). For the MC method we will use 6000 replications with 95% level of confidence based on 5000 bootstrap samples for the estimated point reliability value.

A measurement parameter to measure the accurateness of our method is used, known as the percentage of difference, α. It measures the difference between the Monte Carlo point estimate and the exact reliability value.

$$\alpha = \left| \frac{\text{Estimated value} - \text{Exact value}}{\text{Exact value}} \right| \times 100\% \qquad (1)$$

Figure 3 shows that the confidence interval for Monte Carlo point estimate of terminal reliability envelopes all the exact values for N = 2048 inputs. Table 1 depicts the percentage of difference for terminal reliability is less than 0.024% for N = 16 inputs and 0.134% for N = 2048 inputs. Similar results are shown based on Monte Carlo point estimate for broadcast reliability. The confidence interval of Monte Carlo point estimate covers the exact reliability values for N = 1024 inputs, shown in Figure 4. The percentage of difference for broadcast reliability as in Table 2 is less than 0.210% for N = 128 and 0.100% for N = 1024. The Monte Carlo point estimate confidence interval for network reliability falls below Cheng and Ibe's lower bound as shown in Figure 5 and 6 based on Table 2 and Table 3. But it falls within the bounds of Blake and Trivedi's method. Nevertheless, Monte Carlo point estimate can be used as a source of network reliability estimation as the risk of overestimating the network reliability is lower compared to Cheng and Ibe's method. The percentage of difference for network reliability with N = 16 inputs is less than 0.084% as shown in Table 3.

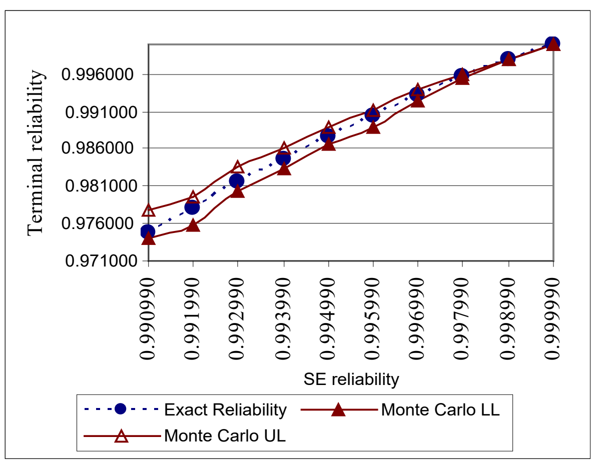

Figure 3. Exact value (Thanawastien &Gunawan's methods) and Monte Carlo point estimate confidence interval of terminal reliability for N= 2048.

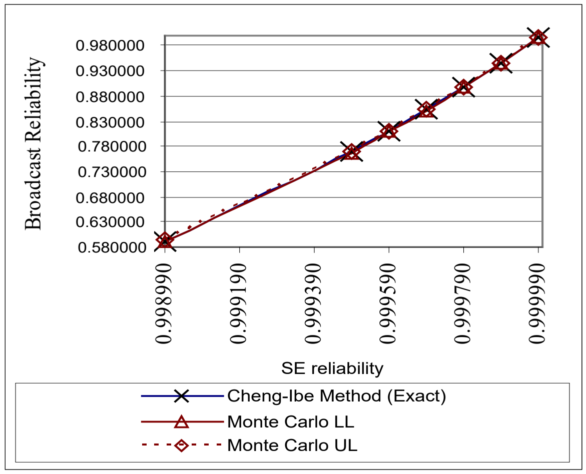

Figure 4. Exact value (Cheng-Ibe method) and Monte Carlo point estimate confidence interval of broadcast reliability for N= 1024.

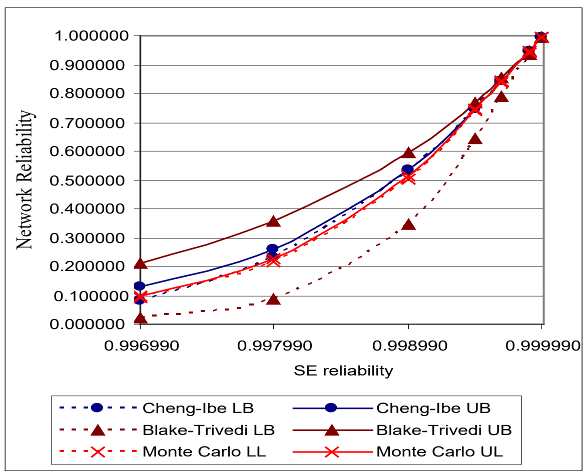

Figure 5. Bounds (Cheng-Ibe and Blake-Trivedi methods) and Monte Carlo point estimate confidence interval for network reliability for N=512.

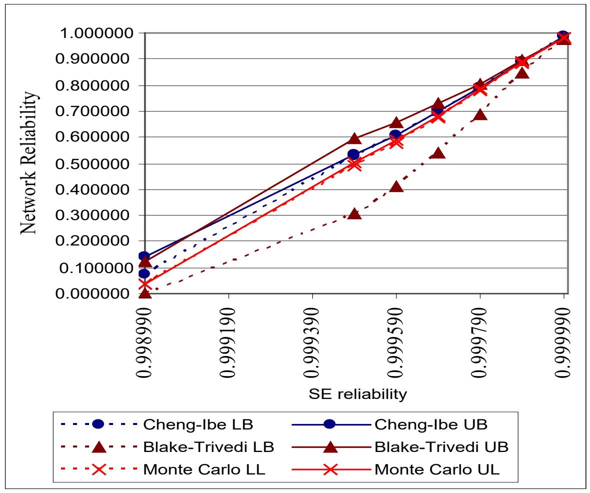

Figure 6. Bounds (Cheng & Ibe's and Blake & Trivedi's methods) and Monte Carlo point estimate confidence interval for network reliability for N=1024.

Table 1. Terminal reliability results for N = 16, 2048.

N	r(t)	Terminal Reliability				α
		Math	Monte Carlo Method			
		Exact	Point Estimate	LL	UL	
16	0.990990	0.981356	0.981406	0.980752	0.981898	0.0051
	0.991990	0.983485	0.983716	0.983224	0.984044	0.0235
	0.992990	0.985599	0.985700	0.985207	0.986029	0.0102
	0.993990	0.987699	0.987522	0.986863	0.988016	0.0179
	0.994990	0.989784	0.989675	0.989180	0.990005	0.0110
	0.995990	0.991854	0.991831	0.991500	0.991996	0.0023
	0.996990	0.993908	0.993823	0.993492	0.993989	0.0086
	0.997990	0.995948	0.995984	0.995984	0.995984	0.0036
	0.998990	0.997972	0.997981	0.997981	0.997981	0.0009
	0.999990	0.999980	0.999980	0.999980	0.999980	0.0000
2048	0.990990	0.974707	0.976005	0.974041	0.977806	0.1332
	0.991990	0.978168	0.977812	0.975844	0.979616	0.0364
	0.992990	0.981479	0.982085	0.980442	0.983564	0.0617
	0.993990	0.984635	0.984723	0.983241	0.986040	0.0089
	0.994990	0.987630	0.987860	0.986705	0.989015	0.0233
	0.995990	0.990457	0.990177	0.989020	0.991169	0.0283
	0.996990	0.993113	0.993161	0.992332	0.993823	0.0048
	0.997990	0.995589	0.995818	0.995486	0.995984	0.0230
	0.998990	0.997880	0.997981	0.997981	0.997981	0.0101
	0.999990	0.999980	0.999980	0.999980	0.999980	0.0000

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Table 2. Broadcast reliability results for N=128, 1024.

N	r(t)	Cheng-Ibe Method	Monte Carlo Method			α
		Exact	Point Estimate	LL	UL	
128	0.990990	0.532779	0.531665	0.528796	0.534349	0.2091
	0.991990	0.573588	0.573420	0.570653	0.575990	0.0293
	0.992990	0.616968	0.616774	0.614137	0.619306	0.0314
	0.993990	0.663012	0.662531	0.660053	0.664896	0.0725
	0.994990	0.711806	0.711006	0.708721	0.713170	0.1124
	0.995990	0.763430	0.764500	0.762832	0.766168	0.1402
	0.996990	0.817951	0.818495	0.817125	0.819728	0.0665
	0.997990	0.875423	0.874923	0.873754	0.875947	0.0571
	0.998990	0.935883	0.936115	0.935647	0.936428	0.0248
	0.999990	0.999350	0.999350	0.999350	0.999350	0.0000
1024	0.998990	0.591132	0.591707	0.590417	0.592897	0.0973
	0.999490	0.768262	0.768718	0.767948	0.769360	0.0594
	0.999590	0.809266	0.809201	0.808391	0.809887	0.0080
	0.999690	0.852334	0.852097	0.851386	0.852665	0.0278
	0.999790	0.897560	0.897111	0.896363	0.897710	0.0500
	0.999890	0.945043	0.945130	0.945130	0.945130	0.0092
	0.999990	0.994882	0.994883	0.994883	0.994883	0.0001

Table 3. Network reliability results for N=16.

N	r(t)	Fard-Gunawan Method	Monte Carlo Method			α
		Actual	Point Estimate	LB	UB	
16	0.990990	0.860108	0.860425	0.8587	0.86201	0.0369
	0.991990	0.875158	0.875888	0.87442	0.87721	0.0834
	0.992990	0.890339	0.89057	0.88923	0.89176	0.0260
	0.993990	0.905644	0.905785	0.90457	0.90684	0.0155
	0.994990	0.921071	0.921091	0.92001	0.92201	0.0022
	0.995990	0.936613	0.936327	0.93539	0.93711	0.0305
	0.996990	0.952267	0.952594	0.95212	0.95291	0.0344
	0.997990	0.968026	0.967997	0.96751	0.96832	0.0030
	0.998990	0.983886	0.983798	0.98347	0.98396	0.0089
	0.999990	0.999840	0.99984	0.99984	0.99984	0.0000

6. Conclusion

In this paper, it is shown that Monte Carlo method with stratified sampling is capable of providing a good estimation on MIN reliability in general. Low percentage of difference and the coverage of the confidence interval prove the applicability of Monte Carlo method. As the Monte Carlo method is based on randomize sampling, results produced in each run may be different. Therefore, the mean of several runs can be used to have a better approximation value.

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Biography

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