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A Two-Stage Robust Optimization Approach for Prepositioning Logistics Items in Disaster Management

Levent Eriskin
Department of Industrial Engineering
Piri Reis University
Istanbul, 34940, Turkey
leriskin@pirireis.edu.tr

Mumtaz Karatas
Biomedical, Industrial and Human Factors Engineering Department
Wright State University
Dayton, OH, 45435, USA
mumtaz.karatas@wright.edu

Abstract

Natural disasters and emergencies pose significant challenges to communities, causing extensive loss of life, economic disruptions, and material damage. As the frequency, intensity, and cost of such events continue to rise, effective disaster management strategies are essential to mitigate their impact. A critical component of disaster preparedness is the planning of logistics depots, and the prepositioning of relief supplies, which play a vital role in ensuring an efficient and organized response. This study addresses the problem of determining the optimal locations for logistics depots and the prepositioning of relief supplies to enhance disaster preparedness. We propose a two-stage robust optimization approach that accounts for uncertainties in demand and travel distances. Our model aims to minimize total costs associated with installation, procurement, transportation, retention, and deprivation. We solve the two-stage model using a column-and-constraint generation algorithm and demonstrate its application through a computational study focused on the Kartal district of Istanbul, Türkiye, considering a potential earthquake scenario.

Keywords
Disaster Management, Location Analysis, Prepositioning Relief Supplies, Humanitarian Logistics, Two-Stage Robust Optimization.

1. Introduction

Natural disasters and emergencies affect communities by causing extensive loss of life, economic disruptions, and material damage. The substantial costs and consequences of these events remain a major concern for many countries. As natural disasters grow more frequent, intense, and costly, the number of people impacted continues to increase at a rapid pace (De Haen and Hemrich 2007). To mitigate the impact of disasters, governments and organizations implement disaster management and recovery actions. Disaster management encompasses all activities carried out before, during, and after a disaster. Since these activities are interconnected and interdependent, they must be coordinated through a holistic approach. Therefore, the concept of integrated disaster risk management (IDRM) has become an important model and strategy for many countries in contemporary disaster management (Sandoval et al., 2023). In particular, IDRM is a comprehensive approach developed for reducing disaster risks by integrating mitigation, preparedness, response, and recovery efforts across multiple sectors and stakeholders (Zhang et al., 2006). The first two phases in this cycle happen before an emergency whereas the last two after it (Neal, 1997).

The main challenge in disaster response is the inherent uncertainty regarding the location, timing, and magnitude of an event. This unpredictability significantly complicates the planning and implementation of disaster preparedness strategies. Effective disaster preparedness not only facilitates an organized response but also ensures that essential resources and support systems are in place for those affected. One critical aspect of this is the planning of temporary shelters, logistic depots and prepositioning of logistics items, which serve play a critical role in mitigating the effects in the aftermath of a disaster.

Once a disaster such as earthquake occurs, affected people are evacuated to temporary shelters. Temporary shelters provide basic services such as healthcare and sanitation and goods such as water, food, clothing to the affected people. Hence, an important part of the humanitarian aid planning for the response phase starting from the preparation phase of disaster management is the planning of logistic requirements that will enable the affected people to sustain their lives temporarily. In general, this planning includes determining the inventory levels of logistic items and the locations of logistics depots where they will be stocked and determining which logistics depot will meet the needs of which demand point. In this planning, the temporary shelters are considered to be demand points, hence, it is assumed that their locations and the number of people to be evacuated to each shelter is known a priori. While it is beyond the scope of this paper, the shelter location-allocation planning is another important decision problem to be addressed in the preparedness phase of the disaster management. Shelter locations and the volume of demand directly impacts the prepositioning strategy. Since the number of affected people depends on the severity of the disaster, the demand volume in each shelter involves uncertainty.

A large body of literature addressing the aforementioned challenges in disaster management has adopted mathematical modeling and optimization methods. To address parameter uncertainties, such as uncertainty in disaster severity or demand, two main approaches are employed: (*i*) Stochastic optimization and (*ii*) Robust optimization. Stochastic optimization methods rely on the assumption that the probability distributions of uncertain parameters are known. However, in scenarios such as natural disasters, it is often impossible to determine the theoretical or empirical probability distributions of uncertain parameters, such as the severity of the disaster. In such cases, robust optimization methods prove to be highly effective.

In this study we address the problem of determining the locations of logistic depots and prepositioning of logistics items to improve the preparedness phase. In particular, our main ambition is to develop a robust optimization approach which accounts for the demand and travel distance uncertainties to determine the locations of logistics depots, the optimal quantity and type of materials to be stocked at each depot, and the optimal allocation of logistics items from depots to shelters. For this purpose, we first propose a deterministic model which assumes that demand and travel distances between shelters and candidate logistics depots are known. As the next step, we develop a two-stage robust optimization model which incorporates uncertainty in demand and distance. The first stage of the robust formulation is the master problem, which is solved for a subset of scenarios, each representing a possible value of uncertain parameters, with the objective of minimizing pre-disaster costs. The second stage, or sub-problem, incorporates the values of the uncertain parameters as constraints in each iteration, aiming to minimize post-disaster costs. We solve this two-stage model using a column-and-constraint generation algorithm. We then demonstrate the results of the proposed approach on a computational study for the Kartal district of Istanbul, Turkiye considering a possible earthquake.

The organization of the paper is as follows: Section 2 presents a literature review on the pre-positioning of relief supplies and logistics warehouse location. Section 3 outlines the preliminaries and provides a formal problem definition. In Section 4, we introduce the deterministic and robust formulations of the problem. Section 5 reports the numerical results for the Kartal, Istanbul case study. Finally, Section 6 concludes the paper with a discussion on future directions.

2. Literature Review

In prepositioning, two key considerations typically arise: the selection of depot locations for storing supplies and assets, and the allocation of these resources to the established depots. As also indicated by Sabbaghtorkan et al. (2020), our literature review reveals that studies on the prepositioning of earthquake relief supplies can be categorized into three main groups: (*i*) location studies, (*ii*) allocation studies, and (*iii*) location-allocation studies.

Location studies focus on determining the most suitable sites for distribution centers without incorporating inventory decisions for those centers. Most recent and notable studies adopting a location-focused perspective include Charles et al. (2016), Stauffer et al. (2016), Cavdur et al. (2016), Elluru et al. (2017), Burkart et al. (2017), Chapman and Mitchell (2018), and Horner et al. (2018). These studies typically consider three different objective functions: (*i*) facility-oriented, (*ii*) transportation-oriented, and (*iii*) demand-oriented. Facility-oriented approaches, such as Chapman and Mitchell (2018), primarily aim to minimize facility operating costs. Transportation-oriented studies focus on minimizing either transportation distance, as seen in Elluru et al. (2017), or transportation costs, as in Burkart et al. (2017) and Horner et al. (2018), to improve the efficiency of humanitarian aid delivery. Demand-oriented studies, including Cavdur et al. (2016) and Burkart et al. (2017), seek to maximize the amount of demand met.

Studies on resource allocation aim to determine the type and quantity of supplies (such as food, water, blankets, and medical equipment) to be stocked at distribution centers with predetermined locations and numbers. Examples of such studies include Kelle et al. (2014), Zhan et al. (2014), Garrido et al. (2015), Alem et al. (2016), Morrice et al. (2016), and Dufour et al. (2018). The most common objective functions in this category include (*i*) minimizing pre-positioning costs of supplies (Kelle et al., 2014), (*ii*) minimizing distribution costs (Garrido et al., 2015), and (*iii*) minimizing the costs associated with unmet demand and excess supplies (Morrice et al., 2016).

Integrated location-allocation studies, such as those by Tofighi et al. (2016), Paul and MacDonald (2016), Chowdhury et al. (2017), Dalal and Üster (2017), Klibi et al. (2018), and Ni et al. (2018), simultaneously determine both the optimal locations of distribution centers and the inventory levels of humanitarian supplies to be stocked in these centers. The most common objective functions in this category include (*i*) minimizing setup costs (Paul and MacDonald, 2016), (*ii*) minimizing pre-positioning costs of materials (Tofighi et al., 2016; Ni et al., 2018), and (*iii*) minimizing inventory holding costs (Chowdhury et al., 2017).

Our review of the literature also reveals that, in terms of solution methods, the majority of studies adopt stochastic optimization approaches due to the inherent uncertainty associated with disasters. Examples of such studies include Campbell and Jones (2011), Chakravarty (2014), and Paul and McDonald (2016). Additionally, studies such as Duran et al. (2011), Hong et al. (2015), Renkli and Duran (2015), and Tofighi et al. (2016) have utilized two-stage stochastic programming, i.e., first-stage decisions made prior to the occurrence of an uncertain event while accounting for potential second-stage decisions, and the second-stage decisions are adjusted accordingly once the uncertainty is resolved. However, these approaches face challenges due to uncertainties in parameters such as the number of affected individuals and transportation times, which are influenced by the unknown severity of events like earthquakes. The lack of data on the probability distributions for these parameters further complicates the application of stochastic methods.

In contrast, the use of robust optimization methods for solving humanitarian aid problems is less common compared to stochastic optimization methods. Among the limited number of studies, Zokaee et al. (2016) proposed a robust optimization model for a three-stage humanitarian logistics chain involving suppliers, distribution centers, and demand points, considering demand, supply availability, and costs as uncertain parameters. Similarly, Ni et al. (2018) developed a two-stage robust optimization model for logistics warehouse selection, inventory level determination, and distribution of a single type of humanitarian aid material. Velasquez et al. (2020) also employed a two-stage robust optimization model for pre-positioning humanitarian supplies, extending the approach to address multiple consecutive disasters and post-disaster supply considerations.

To the best of our knowledge, none of the existing robust optimization models adopt a holistic modeling perspective that accounts for both demand uncertainty and the vulnerability of transportation lines following a disaster, such as an earthquake, when optimizing the prepositioning of depots and relief items.

3. Preliminaries and Problem Definition

This study addresses the problem of prepositioning logistics items in disaster management. Prepositioning decisions belong to the preparedness phase of disaster management; hence, these decisions are made before a disaster occurs. As emphasized in Section 1, these decisions are closely related to shelter location-allocation decisions which are also made in the preparedness phase of the disaster management. Temporary shelters are the demand points for the humanitarian logistic items and the locations of the shelters as well as the demand volume are the parameters for the prepositioning problem.

To that end, in this problem, we would like to determine which of the candidate logistics depots should be installed, how much of which material should be stocked in which depot, and how much logistic item should be transported from which depot to which shelter in a way to minimize the total costs. In this problem, we consider five different cost items: (*i*) Installation of logistics depots, (*ii*) Procurement of logistic items, (*iii*) Transportation of logistic items, (*iv*) Retention of logistic items, and (v) Deprivation of logistic items.

We consider a set of shelter locations $i \in I$ as demand points for the logistic items and a set of candidate locations $j \in J$ for logistics depots. The traveling distance between shelter $i \in I$ and candidate logistics depot location $j \in J$ is $dist_{ij}$. The set of logistic items is defined as $k \in K$. The procurement cost of each logistic item $k \in K$ is c_k while transportation cost per kilometer is t_k. The unit volume required for storage of logistic item $k \in K$ is v_k. The demand of shelter $i \in I$ for logistics item $k \in K$ is known and denoted as d_{ik}. Since the demand for the logistic items inherently involves uncertainty, it is possible that some of the demand may not be met, or we may have a remaining stock of logistic items after the demand is satisfied. In this respect, the cost of not meeting the demand for logistic item $k \in K$ is p_k and retention cost of unused logistic item $k \in K$ is h_k. Each candidate logistics depot location $j \in J$ has a storage capacity of V_j while the cost of establishing a logistics depot at location $j \in J$ is f_j. Since the prepositioning decisions are made in the preparedness phase of disaster management, we assume that there is no restriction on the availability of logistic items. Moreover, it is assumed that locations and number of people evacuated to each shelter are known a priori and determined based on a certain disaster scenario.

4. Formulations

In this section, we first provide the deterministic model where we assume that the logistic demand of each shelter and the traveling distances between shelters and candidate logistics depots are known deterministically. Then, we present our two-stage robust model that accounts for the uncertainty inherent in the logistic item demands of shelters and the transportation distance between shelters and candidate logistics depots. We define the deterministic model below.

Sets and Indices
$i \in I$: Set of shelter locations
$j \in J$: Set of candidate logistics depot locations
$k \in K$: Set of humanitarian logistics items

Parameters
f_j: Cost of installing a logistics depot at location $j \in J$
d_{ik}: Demand of shelter $i \in I$ for logistics item $k \in K$
c_k: Unit cost of logistics item $k \in K$
t_k: Transportation cost per kilometer of logistics item $k \in K$
v_k: Unit volume required for storage of logistics item $k \in K$
h_k: Retention cost of unused (retained) logistics item $k \in K$
p_k: Cost of not meeting the demand of logistics item $k \in K$
V_j: Total volume (capacity) of a logistics depot to be established at location $j \in J$
$dist_{ij}$: The distance between shelter $i \in I$ and candidate logistics depot location $j \in J$

Decision Variables
$b_j = \begin{cases} 1, \text{if a logistics depot is established at location } j \in J \\ 0, \text{otherwise} \end{cases}$
x_{kj}: Amount of logistics item $k \in K$ to be stored at location $j \in J$
y_{ijk}: Amount of logistics item $k \in K$ to be sent to shelter $i \in I$ from location $j \in J$
e_{ki}: Amount of shelter $i \in I$'s demand for logistics item $k \in K$ that cannot be met
o_{kj}: Amount of logistics item $k \in K$ remaining in the logistics depot at location $j \in J$

The Model (D)

$$Min \quad \sum_{j\in J} f_j b_j + \sum_{k\in K}\sum_{j\in J} c_k x_{kj} + \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_k dist_{ij}\, y_{ijk} + \sum_{k\in K}\sum_{i\in I} p_k e_{ki} + \sum_{k\in K}\sum_{j\in J} h_k o_{kj} \tag{1}$$

$$st \quad \sum_{j\in J} y_{ijk} + e_{ik} \geq d_{ik}, \qquad \forall i \in I, \forall k \in K \tag{2}$$

$$x_{kj} - \sum_{i\in I} y_{ijk} \geq o_{kj}, \qquad \forall j \in J, \forall k \in K \tag{3}$$

$$\sum_{k\in K} v_k x_{kj} \leq V_j b_j, \qquad \forall j \in J \tag{4}$$

$$y_{ijk} \geq 0, \qquad \forall i \in I, \forall j \in J, \forall k \in K \tag{5}$$

$$x_{kj} \geq 0, \qquad \forall j \in J, \forall k \in K \tag{6}$$

$$o_{kj} \geq 0, \qquad \forall j \in J, \forall k \in K \tag{7}$$

$$e_{ki} \geq 0, \qquad \forall i \in I, \forall k \in K \tag{8}$$

$$b_j \in \{0,1\}, \qquad \forall j \in J \tag{9}$$

In model (D), the objective function minimizes the installation costs of the logistics depots as well as humanitarian logistics item procurement, transportation, retention, and deprivation costs. Constraint (2) ensures that the demands of shelters are met by taking into account the amount that cannot be met, and constraint (3) ensures that the items to be transported from the installed logistics depots to the shelters do not exceed the level of items stocked in the depots, constraint (4) provides that the total items to be stocked in the logistics depots do not exceed the capacity of the depots installed.

As emphasized previously, model (D) assumes that the logistic item demands of shelters and the traveling distances between shelters and candidate logistics depots are known with certainty. However, the number of people who will be affected by the disaster depends on the severity of the disaster. Likewise, travel distances between shelters and candidate logistics depots may increase due to partial road closures caused by the disaster. To that end, both problem parameters are subject to uncertainty. While it is beyond the scope of this paper, by solving the shelter location-allocation problem, the locations of the shelters and the number of people to be evacuated to these shelters are determined with respect to different disaster scenarios. For a detailed discussion regarding the subject and an application example, the reader is referred to Eriskin and Karatas (2024). In this context, assuming that logistic item demands of shelters, that is a function of the number of people evacuated to the shelters, and the traveling distances between shelters and candidate logistics depots as well as their variability are known, a two-stage robust model has been developed for the logistic item prepositioning problem.

Depending on the severity of the disaster, the traveling distance between the shelters and the candidate logistics depot locations may vary depending on (*i*) the number of affected people to be evacuated to shelters (demand for logistic items) and (*ii*) the damage caused by the disaster to the transportation lines. Let $D = \{d_{ik}, i \in I, k \in K\}$ and $Z = \left\{dist_{ij}, i \in I, j \in J\right\}$ and the uncertainty associated with these values is represented by the set U. In this framework, the values of the uncertain parameters d_{ik} and $dist_{ij}$ depending on the uncertainty set U can be defined as follows:

Additional Parameters
$\underline{d}_{ik}$: Minimum demand of shelter $i \in I$ for logistics item $k \in K$
$\hat{d}_{ik}$: Maximum deviation of demand of shelter $i \in I$ from $\underline{d}_{ik}$ for logistics item $k \in K$
$\underline{dist}_{ij}$: Shortest distance from shelter $i \in I$ to location $j \in J$
$\widehat{dist}_{ij}$: Maximum deviation of shelter $i \in I$ from the shortest distance $\underline{dist}_{ij}$ to location $j \in J$
Γ^{τ}: Total uncertainty budget of shelter demands, $\Gamma^{\tau} \in [0, |I|]$
Γ^{ξ}: Total uncertainty budget of the distance between shelters and logistics depots, $\Gamma^{\xi} \in [0, |I||J|]$

Additional Decision Variables
τ_{ik}: Uncertainty level of shelter $i \in I$'s demand for logistics item $k \in K$
ξ_{ij}: Uncertainty level of the shortest distance of shelter $i \in I$ to location $j \in J$

$$d_{ik} = \underline{d}_{ik} + \tau_{ik}\hat{d}_{ik}, \qquad \forall i \in I, \forall k \in K \tag{10}$$

$$dist_{ij} = \underline{dist}_{ij} + \xi_{ij}\widehat{dist}_{ij}, \qquad \forall i \in I, \forall j \in J \tag{11}$$

$$\sum_{i \in I} \tau_{ik} \le \Gamma^{\tau}, \qquad \forall k \in K \tag{12}$$

$$\sum_{i \in I}\sum_{j \in J} \xi_{ij} \le \Gamma^{\xi} \tag{13}$$

$$\tau_{ik} \in [0,1], \qquad \forall i \in I, \forall k \in K \tag{14}$$

$$\xi_{ij} \in [0,1], \qquad \forall i \in I, \forall j \in J \tag{15}$$

As seen from the above formulation, uncertainty budgets are also defined for the uncertainty levels τ_{ik} for each shelter and ξ_{ij} for the distances between shelters and candidate logistics depots (Constraints (12) and (13)). As remarked by Bertsimas and Thiele (2006), the rationale behind the use of uncertainty budgets is that not all uncertain parameters will simultaneously deviate from their estimated nominal values and that some values will be realized more than the estimated nominal value and some less than this value. In the aftermath of a disaster, shelters will accommodate affected people from multiple regions, the number of which will be uncertain for each region depending on the severity of the disaster. In addition, the level of damage to roads after a disaster, i.e. the level of deviation from their nominal values, may be different from each other. To that end, it is considered reasonable to use the uncertainty budget for both uncertainty levels.

In prepositioning planning, the location and number of logistics depots and the number of logistic items to be stocked in these depots are determined before the disaster occurs (first stage), and once the disaster occurs (second stage), the stocked items are distributed to temporary shelters. In the light of these information, the two-stage robust model for the prepositioning problem can be defined as follows:

The Model (TSRM)

$$\min_{X,B} \sum_{j\in J} f_j\, b_j + \sum_{k\in K}\sum_{j\in J} c_k x_{kj} + \max_{(D,Z)\in U} \left\{ \min_{Y,E,O} \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_k dist_{ij}\, y_{ijk} + \sum_{k\in K}\sum_{i\in I} p_k e_{ki} + \sum_{k\in K}\sum_{j\in J} h_k o_{kj} \right\} \tag{16}$$

st. Constraints (2)-(9) (17)

In the defined model, b_j and x_{kj} are the first-stage decision variables, y_{ijk}, e_{ki} and o_{kj} are the second-stage decision variables. The first part of the objective function represents the first-stage costs (before the disaster occurs) and the second part represents the second-stage costs (after the disaster occurs).

The uncertainty set U is a continuous, non-finite, polyhedral set. Therefore, it is not possible to linearize the non-linear second term in the objective function of the (TSRM) model by explicitly enumerating all scenarios. Let $L = \{(D_1, Z_1), \dots, (D_l, Z_l) | L \subset U\}$ define a discrete subset L of the values that the uncertain parameters may take on with respect to the disaster scenarios. Let the decision variables (Y_l, E_l, O_l) of the second stage problem take their values (Y_l, E_l, O_l) for each possible state $l \in L$ (the set of values that the uncertain parameters can take). In this case, the (TSRM) model for the set L, which is a discrete subset of the polyhedral uncertainty set U, can be transformed into the following linear form:

The Model (MP)

$$Min \quad \sum_{j\in J} f_j\, b_j + \sum_{k\in K}\sum_{j\in J} c_k x_{kj} + \eta \tag{18}$$

st Constraints (4), (6) and (9)

$$\eta \geq \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_k dist_{ij}^l\, y_{ijk}^l + \sum_{k\in K}\sum_{i\in I} p_k e_{ki}^l + \sum_{k\in K}\sum_{j\in J} h_k o_{kj}^l, \qquad \forall l \in L \tag{19}$$

$$\sum_{j\in J} y_{ijk}^l + e_{ki}^l \geq d_{ik}^l, \qquad \forall i \in I, \forall k \in K, \forall l \in L \tag{20}$$

$$x_{kj} - \sum_{i\in I} y_{ijk}^l \geq o_{kj}^l, \qquad \forall j \in J, \forall k \in K, \forall l \in L \tag{21}$$

$$o_{kj}^l \geq 0, \qquad \forall j \in J, \forall k \in K, \forall l \in L \tag{22}$$

$$e_{ki}^l \geq 0, \qquad \forall i \in I, \forall k \in K, \forall l \in L \tag{23}$$

$$y_{ijk}^l \geq 0, \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L \tag{24}$$

It is not possible to solve the mixed integer linear program (MP), since there is no finite number of values that the uncertain parameters can take in the set U. Many methods have been proposed in the literature to solve such optimization problems by taking advantage of their special structure. One of them is the column and constraint generation method (Zeng and Zhao, 2013). In this method, the master (first stage) problem is solved with a subset of the scenario set (the values that the uncertain parameters can take in the set U), and the generated sub (second stage) problem adds the constraint scenarios (the values of the uncertain parameters) to the scenario set at each iteration. For a model with a minimization objective function, the master problem gives a lower bound on the optimal solution, while the sub-problem yields the upper bound. At each step, more restrictive scenarios are generated, and the master problem is solved with this set of scenarios. The algorithm ends when the gap between the lower and upper bound falls below a certain threshold. In this context, in this step, the problem will be solved by the column and constraint generation method.

If the (MP) model is defined for a set of L scenarios consisting of a subset of these scenarios, instead of all possible scenarios, the resulting model provides a valid relaxation for the original (TSRM) model. Since the (TSRM) model is a minimization problem, the (MP) model to be solved for the subset L yields a lower bound for the (TSRM) model. In this framework, for given values of b_j and x_{kj}, which represent the decisions in the first stage of the problem, the important (restrictive) scenarios in the uncertainty set U can be obtained by solving the following subproblem:

The Model (SP)

$$\max_{(D,Z)\in U} \left\{ \min_{Y,E,O} \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_k dist_{ij}\, y_{ijk} + \sum_{k\in K}\sum_{i\in I} p_k e_{ki} + \sum_{k\in K}\sum_{j\in J} h_k o_{kj} \right\} \tag{25}$$

$$st \quad \sum_{j\in J} y_{ijk} + e_{ik} \geq d_{ik}, \qquad \forall i\in I, \forall k\in K \tag{26}$$

$$x_{kj} - \sum_{i\in I} y_{ijk} \geq o_{kj}, \qquad \forall j\in J, \forall k\in K \tag{27}$$

$$y_{ijk} \geq 0, \qquad \forall i\in I, \forall j\in J, \forall k\in K \tag{28}$$

$$o_{kj} \geq 0, \qquad \forall j\in J, \forall k\in K \tag{29}$$

$$e_{ki} \geq 0, \qquad \forall i\in I, \forall k\in K \tag{30}$$

By solving the model (SP), the values that give the largest second-stage cost (η) are calculated and returned to the model (MP). As the model (MP) will find the best solution with an increasing set of scenarios in each iteration, it will generate non-decreasing lower bounds. However, the model (SP) is a two-stage (max-min) model. By taking the dual of the minimization problem in model (SP) and defining the uncertain parameters (d_{ik} and $dist_{ij}$) as functions of the uncertainty level variables (τ_{ik} and ξ_{ij}), the problem can be transformed into the following form:

The Model (SP')

$$max \quad \sum_{i\in I}\sum_{k\in K} (\underline{d}_{ik} + \tau_{ik}\hat{d}_{ik})\lambda_{ik} - \sum_{k\in K}\sum_{j\in J} x_{kj}\gamma_{kj} \tag{31}$$

$$st \quad \lambda_{ik} \leq p_k, \qquad \forall i\in I, \forall k\in K \tag{32}$$

$$-\gamma_{kj} \leq h_k, \qquad \forall j\in J, \forall k\in K \tag{33}$$

$$\lambda_{ik} - \gamma_{kj} \leq t_k\left(\underline{dist}_{ij} + \xi_{ij}\widehat{dist}_{ij}\right), \qquad \forall i\in I, \forall j\in J, \forall k\in K \tag{34}$$

$$\lambda_{ik} \geq 0, \qquad \forall i\in I, \forall k\in K \tag{35}$$

$$\gamma_{kj} \geq 0, \qquad \forall j\in J, \forall k\in K \tag{36}$$

$$\tau_{ik} \in [0,1], \qquad \forall i\in I, \forall k\in K \tag{37}$$

$$\xi_{ij} \in [0,1], \qquad \forall i\in I, \forall j\in J \tag{38}$$

Constraints (12) and (13)

In the single-level model (SP'), λ_{ik} and γ_{kj} are dual variables. Due to the nonlinear term in the objective function (the product of τ_{ik} and λ_{ik}), the model is nonlinear. This model can be linearized based on McMormick envelope relaxation (McCormick, 1976). Let us first define a non-negative continuous variable δ_{ik} to represent the expression τ_{ik} and λ_{ik} in the objective function. Let M_{ik} be the upper bound value for λ_{ik}. Consider the following propositions:

$$\tau_{ik} = 0 \Rightarrow \delta_{ik} = 0$$

$$\tau_{ik} = 1 \Rightarrow \delta_{ik} = \lambda_{ik}$$

The linearized version of the model (SP') that ensures the above propositions can be defined as follows.

The Model (SP")

$$max \quad \sum_{i\in I}\sum_{k\in K} \underline{d}_{ik}\lambda_{ik} + \sum_{i\in I}\sum_{k\in K} \hat{d}_{ik}\delta_{ik} - \sum_{k\in K}\sum_{j\in J} x_{kj}\,\gamma_{kj} \tag{39}$$

$$st \quad \delta_{ik} \le M_{ik}\tau_{ik}, \quad \forall i \in I, \forall k \in K \tag{40}$$

$$\delta_{ik} \le \lambda_{ik}, \quad \forall i \in I, \forall k \in K \tag{41}$$

$$\delta_{ik} \ge \lambda_{ik} - M_{ik}(1 - \tau_{ik}), \quad \forall i \in I, \forall k \in K \tag{42}$$

$$\delta_{ik} \ge 0, \quad \forall i \in I, \forall k \in K \tag{43}$$

Constraints (32)-(38)

The problem consisting of master (MP) and sub (SP") models can be solved iteratively by applying the column and constraint generation algorithm given in Figure 1.

1. Set Upper Bound $(UB) = \infty$, Lower Bound $(LB) = -\infty, n = 1, \epsilon \ge 0$. Identify a parameter set $(D_n, Z_n) \in U$ and set $L = \{(D_n, Z_n)\}$.
2. Solve (**MP**) to obtain optimal solution (X_n^*, B_n^*) and objective function value $Z_{MP}(L)$. Update $LB \leftarrow Z_{MP}(L)$.
3. Solve $(\mathbf{SP}'')$ to obtain optimal solution (D_n^*, Z_n^*) and objective function value $Z_{\boldsymbol{SP'}}(X_n^*, B_n^*)$. Update $UB \leftarrow \min\{UB, \sum_{j\in J} f_j b_j + \sum_{k\in K}\sum_{j\in J} c_k x_{kj} + Z_{\boldsymbol{SP'}}(X_n^*, B_n^*)\}$.
4. If $(UB - LB) \le \epsilon$, return (X_n^*, B_n^*) and end the procedure. Otherwise, update $L \leftarrow L \cup \{(D_n^*, Z_n^*)\}, n \leftarrow n + 1$. Go back to step 2.

Figure 1. Pseudo-code for the column and constraint generation algorithm

5. Case Study

We conducted a computational study for the Kartal district of Istanbul considering a possible earthquake. Having 20 subdistricts, Kartal district is on the Anatolian side of Istanbul with an area of 38.54 km^2 and a population of 425,000. Being responsible for the coordination of humanitarian relief activities in Turkey, the AFAD (Disaster and Emergency Management Directorate) determines 15 candidate logistic depot locations in Kartal with different storage capacities. Capacities of these candidate depot locations and installation costs are given in Table 1.

Table 1. Capacities and installation costs of candidate depot locations

Cand.Dep. ID	Capacity (m^3)	Install. Cost (TL)	Cand. Depot ID	Capacity (m^3)	Install. Cost (TL)
1	4,200	4,290,000	13	10,500	10,725,000
2	7,875	8,043,750	14	8,750	8,937,500
3	4,375	4,468,750	16	5,250	5,362,500
4	10,500	10,725,000	18	7,875	8,043,750
6	4,375	4,468,750	20	5,250	1,787,500
10	17,500	17,875,000	23	26,250	26,812,500
11	5,250	5,362,500	25	10,500	10,725,000
12	10,937	11,171,875			

Firstly, the number of people to be evacuated to the shelters from each subdistrict is estimated with the shelter demand prediction methodology and the evacuation plan is determined by solving the shelter location-allocation problem. Details of the shelter demand prediction methodology, the model developed for the shelter location-allocation problem, and details of the evacuation plan can be found in Eriskin and Karatas (2024). As discussed in Eriskin and Karatas (2024), the model opens 10 shelters within the Kartal district and allocates affected people to these shelters. In this study, these 10 shelters are demand points for the logistic items and the number of people allocated to each shelter drives the demand volume.

In line with the practices adopted by AFAD, in this study, the following logistic items are considered for prepositioning: (*i*) medical aid kit, (*ii*) water, (*iii*) tent and tent material, (*iv*) hygiene kit, and (*v*) dry food. The unit procurement and transportation costs as well as volumes of logistic items to meet the needs of one person for one week are given in Table 2. The cost of not meeting the demand for logistic item (p_k) is assumed to be four times the unit procurement cost while the retention cost of unused (retained) logistic item (h_k) is assumed to be one-tenth of the unit procurement cost.

Table 2. Humanitarian logistic items to be planned for the affected people

Item ID	Logistic Item	Volume (m^3)	Procurement Cost (TL)	Transportation Cost (TL/km)
1	Medical aid kit (Note-1)	0.0014	165	0.000158
2	Water (Note-2)	0.021	210	0.00237
3	Tent and tent material (Note-3)	0.17	22,000	0.019185
4	Hygiene kit (Note-4)	0.0014	40	0.000158
5	Dry food (Note-5)	0.042	500	0.00474

Note-1: The medical kit (Red Crescent first aid kit) recommended by the American Red Cross. It is assumed that the medical kits will only be used for the lightly wounded since the seriously wounded will be treated in the hospital and there will be a need for one for each tent.
Note-2: 6 packs of half a liter per day, 42 half liters of water per week. 42 half liters of water is planned for each person.
Note-3: 16.5 m^2 shelter tent for four people and materials inside the tent (https://www.kizilaycadirtekstil.com.tr). One tent is planned for every four people.
Note-4: Activex hygiene kit. One is planned for each person.
Note-5: 7-day dry food kit consisting of tuna fish, beans, and leaf rolls. One kit is planned for each person.

The demand prediction methodology and shelter location-allocation model developed by Eriskin and Karatas (2024) determine the number of people to be evacuated to each shelter and the variability associated with these values. Using these predictions, the logistic item demand of each shelter (d_{ik}) is estimated. The estimated logistic item demands of shelters (d_{ik}) are assumed to correspond to the minimum demands of shelters ($\underline{d}_{ik}$) in the two-stage robust model, while the maximum deviations in demands ($\hat{d}_{ik}$) are estimated based on the variability of the number of evacuees to each shelter. To that end, the logistic item demands of shelters (d_{ik}) are given in Table 3.

Table 3. Logistic item demands of shelters

Logistic Item	Shelter ID									
	5	7	8	9	15	17	19	21	22	24
Medical aid kit	2527	1229	3189	1082	1583	3065	2524	1401	1180	1408
Water	10105	4916	12756	4325	6332	12257	10096	5604	4718	5630
Tent and tent material	2527	1229	3189	1082	1583	3065	2524	1401	1180	1408
Hygiene kit	10105	4916	12756	4325	6332	12257	10096	5604	4718	5630
Dry food	10105	4916	12756	4325	6332	12257	10096	5604	4718	5630

It is assumed that the logistic items will be transported from logistics depots to the shelters by road. The "Open-Source Routing Machine (OSRM)" API, which combines "OpenStreetMap" based complex routing algorithms with road network data, is used to obtain vehicle travel distances between candidate logistic depots and shelters ($dist_{ij}$). Providing the interface between the R program and OSRM, "Package OSRM", is used to extract data from this application in an efficient way. To account for the road vulnerability, it is assumed that the distances between candidate logistic depot locations and shelters ($dist_{ij}$) correspond to the shortest distances from shelters to candidate locations ($\underline{dist}_{ij}$) in the two-stage robust model. To calculate the vulnerability of the roads, the narrow road ratios (NRR) in the districts provided in the JICA Report (2002) are used. The NRRs of the districts are used to calculate the road vulnerabilities (RV) of the routes between demand points (temporary shelters) and candidate logistics depots, as applied in studies such as Renkli and Duran (2015) and Başkaya et al. (2017). For example, the NRR for the route between the shelter $i \in I$ and the candidate location $j \in J$ passing through the districts listed from 1 to m is computed by the formula $RV_{ij} = (NRR_1 + \cdots + NRR_m)/m$. Then, the maximum deviation from the minimum values of the distances between shelters and candidate logistics depots after an earthquake is calculated with the following formula:

$$\widehat{dist}_{ij} = \underline{dist}_{ij}\left(\frac{1}{1 - RV_{ij}}\right) - \underline{dist}_{ij} \tag{44}$$

All models are implemented in GAMS and R and solved with CPLEX 12.5. Runs are performed on a computer having an Intel Core i5-8265U 2.40 GHz processor and 8 GB of RAM. The total uncertainty budget of shelter demands (Γ^{τ}) is set to 10 and the total uncertainty budget of the distance between shelters and logistics depots (Γ^{ξ}) is set to 150. The solution time of the Kartal instance with the column and constraint generation algorithm is 104 seconds. The two-stage robust model for the Kartal instance is solved with 3 iterations ($|L| = 3$), each corresponding to a realization of a different scenario. The logistic depots to be installed and the number of logistic items to be stored in these depots at each iteration are given in Table 4.

Table 4. Logistic depots to be installed and the number of logistic items to be stored in these depots at each iteration

Iteration	**Depots Installed**	**Logistic Items Stored Depot (Number of Items Stored)**				
		1	**2**	**3**	**4**	**5**
1	3-11	3 (8,528) 11 (10,660)	3 (34,109) 11 (42,630)	3 (8,528) 11 (10,660)	3 (34,109) 11 (42,630)	3 (34,109) 11 (42,630)
2	1-6-16	1 (3,932) 6 (14,438) 16 (10,838)	1 (24.354) 6 (55,445) 16 (37,009)	1 (6.997) 6 (5,490) 16 (16,721)	1 (15,726) 6 (57,742) 16 (43,341)	1 (27,983) 6 (51,817) 16 (37,009)
3	6-11-20	6 (9,607) 11 (7,773) 20 (11,828)	6 (57,742) 11 (31,084) 20 (27,983)	6 (8,342) 11 (7,773) 20 (13,094)	6 (57,742) 11 (31,084) 20 (27,983)	6 (39,286) 11 (31,084) 20 (46,436)

Iteration 3 solution corresponds to the optimal solution of the problem. The objective function value of the model (MP) is 750,201,280.2 TL and the solution recommends installing logistics depots 6, 11, and 20. The shelters that will served by these depots are presented in Table 5 while the locations of the depots to be installed and temporary shelters of Kartal district are shown in Figure 2.

Table 5. The shelters that will be served by the depots to be installed

Depot ID	**Shelter ID**
6	5-7-8-9-15-17-19-21-22
11	5-7-15-17-24
20	5-7-8-9-15-17-19-21-22-24

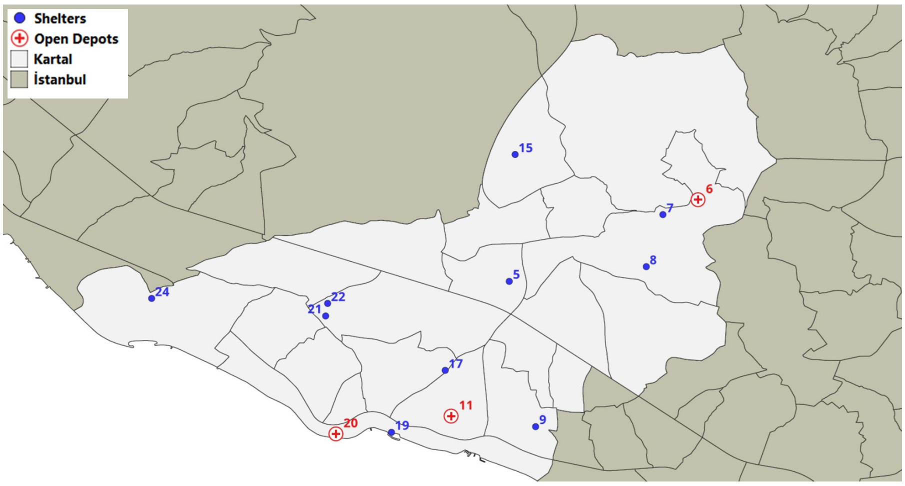

Figure 2. Locations of logistics depots to be installed and temporary shelters of Kartal district

In the optimal solution obtained for the Kartal instance, installation, procurement, transportation, retention, and deprivation costs are realized as (*i*) 11,618,750 TL, (*ii*) 735,002,070 TL, (*iii*) 3,580,460 TL, (*iv*) 0 TL and (*v*) 0 TL, respectively. The biggest cost item among these costs is the logistic item procurement cost. Since the costs of not meeting the logistic demands are relatively high with respect to the procurement costs, all demand is met with the inventory while no inventory is expected to be left once logistic items are distributed to shelters.

7. Conclusion

An important part of humanitarian aid planning for the response phase of disaster management, starting from the preparation phase, is the planning of logistics requirements that will enable the people affected by the disaster to sustain their lives temporarily. In general, this planning involves determining the inventory levels of logistics items and the locations of the logistics depots where they will be stocked, and determining which depot will meet the needs of which demand point.

Addressing the prepositioning problem, we formulate a two-stage robust optimization model for prepositioning relief supplies. The proposed model determines the location of logistics depots and the quantity of logistic items to be prepositioned in these depots. The model takes into account the uncertainty associated with the demand volume for the logistic items and the traveling distances between logistic depots and shelters. We propose a column and constraint generation algorithm to solve the problem efficiently. We also showcase the applicability of the proposed solution approach through a case study targeting earthquake disaster management in Istanbul, Türkiye.

There are two possible future research directions for this study. First, objective functions other than the total cost can be considered in prepositioning decisions. For instance, the utilization of logistic depots and equity in resource distribution between demand points can be considered. To that end, trade-offs inherent in disaster management decisions can be taken into account. Second, it might be interesting to conduct a comparative study to assess the performance of stochastic versus robust optimization models under various disaster scenarios, focusing on trade-offs in computational efficiency and solution quality.

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Biographies

Levent Erişkin received B.S. (1998) in Industrial Engineering from the Turkish Naval Academy, M.S. (2003) in Operations Research from the US Naval Postgraduate School, and Ph.D. (2015) in Industrial Engineering from the Middle East Technical University. He is currently Associate Professor at the Piri Reis University. His research areas include Optimization, Location Theory, Statistical Learning, and Military Operations Research.

Mümtaz Karataş is currently an Associate Professor at the Wright State University. His primary research focuses on the applications of optimization, machine learning, and data analytics to tackle supply chain design and

logistics problems, as well as defense planning problems. He specializes in areas such as facility location and sizing, vehicle/inventory routing, sensor network design, and transportation planning.