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Optimizing Energy-Intensive Heating Processes in Manufacturing Using Bayesian Belief Networks: A Case Study in Thermoforming

Iman Jalilvand, Niloofar Akbarian-Saravi, Bryn Crawford and Abbas S. Milani
Materials and Manufacturing Research Institute
School of Engineering
The University of British Columbia
Kelowna, BC, Canada
iman.jalilvand@ubc.ca, niloofar.akbarian@ubc.ca, bryn.crawford@ubc.ca, abbas.milani@ubc.ca

Bhushan Gopaluni
Data Analytics and Intelligent Systems (DAIS) Lab
Department of Chemical and Biological Engineering
The University of British Columbia
Vancouver, Canada
bhushan.gopaluni@ubc.ca

Abstract

This study investigates the application of Bayesian Belief Networks (BBNs) to optimize thermoforming, an energy-intensive manufacturing process that rapidly shapes thermoplastic composite sheets into precise three-dimensional forms. Thermoforming involves complex interactions among process parameters, presenting ongoing challenges in achieving optimal energy efficiency, temperature stability, and consistent product quality. Further, the process can become highly non-linear with multiple sources of noise, due to the nature of the factory and logistical factors, adding further challenges to effectively predicting the process outcomes. To address this challenge, in this case study a Bayesian Belief Network was developed to capture the inherent uncertainties and intricate relationships among critical process variables. This probabilistic model integrated experimental observations, computational modeling results, and expert-derived insights, enabling a robust and adaptive decision-making support for the process designers. Each of the select key process parameters such as convective Heat Transfer Coefficient (HTC), and heating power, were prioritized based on their influence on the process performance indicators such as the end product quality, energy usage, temperature distribution error, settling time, and stability. Results demonstrated that the BBN framework provides an effective interactive decision support tool capable of continuous model refinements through updating of probabilities as new data became available, while achieving enhanced energy efficiency and process control, thereby also reducing the operational cost and material wastage.

Keywords
Bayesian Belief Networks (BBNs), Thermoforming, Energy Efficiency, Process Optimization, Probabilistic Modeling.

1. Introduction
In pursuit of energy-efficient and high-performance manufacturing, thermoforming has gained significant traction as a cost-effective and rapid method for shaping thermoplastic materials and composites into complex geometries, ideally

with no major defects such as wrinkling (Throne, 2011; Kashani et al., 2017). Industries such as automotive, aerospace, and packaging rely heavily on thermoforming to deliver lightweight, precision-formed parts with minimal material waste. Yet, despite its widespread adoption, the process remains difficult to control and optimize due to the highly interdependent nature of its thermal, mechanical, and material parameters (Yang & Hung, 2004). Minor fluctuations, e.g., in initial sheet temperature, airflow above the sheet (HTC), or heater power can lead to substantial variations in product quality, including warping, thinning, or internal stresses, resulting in process inefficiencies and economic losses (Throne, 2011).

Conventional control strategies often fail to provide sufficient insight or adaptability in real-time decision-making for such complex forming processes, especially in the presence of uncertain and incomplete data. Moreover, black-box machine learning approaches like artificial neural networks (ANNs), although powerful for prediction, often lack interpretability and robustness when applied to material systems or processes with limited or noisy datasets (Liu et al., 2020; Patil et al., 2021; Turan et al., 2022). This creates a pressing need for modeling frameworks that not only capture the probabilistic dependencies among the process parameters, but also offer transparent reasoning to guide the designer and operator decisions under uncertainty. Bayesian Belief Networks (BBNs) have shown great promise in addressing such challenges due to their ability to integrate expert knowledge, empirical data, and simulation results into a unified probabilistic modeling framework (Heckerman, 1997). Their graphical structure enables reasoning across both direct and indirect influences among variables, while supporting inference under partially observed conditions. In manufacturing settings, BBNs have already demonstrated utility in the managerial decision support, process risk management, and fault diagnostics (Haas et al., 2024). However, their application to thermoforming remains relatively scarce, particularly in modeling the intermediate thermal behavior of the system and its downstream impact on energy use and product quality.

This research is motivated by the practical need for an interpretable and robust decision tool in the context of thermoforming process optimization, where trial-and-error approaches may be costly and error-prone (Figure 1). By developing a structured BBN tailored to thermoforming, we hypothesize that a BNN can capture the multi-layered dependencies among key process inputs (e.g., initial temperature, heating power), intermediates (e.g., error, stability), and final outcome (i.e., product quality), offering a pathway to more resilient and energy-aware process control.

1.1 Objective

This case study examines the potential use of BBN to enhance control precision and decision-making adaptability in the high-power heating set-up of a thermoforming process. Namely, by integrating BBN, the *concurrent and interactive* effects of critical process decision (input) parameters are modeled, including the power levels of overhead heating elements set in array, the mechanical properties of a heated acrylic sheet throughout the manufacturing process, the initial ambient temperature, among others. It is shown that selecting optimal levels of these parameters impacts the key process performance indicators (KPIs) such as the total energy consumption, expected error in the eventual temperature distribution over the sheet regions, settling time, process stability, and ultimately product quality. Accordingly, the proposed probabilistic framework can theoretically capture these inter- and intra-dependencies among the input and output parameters in a visual and interactive way, offering a novel dynamic and precise control strategy and decision-making framework for practitioners and designers. Through a sensitivity analysis, dominant process parameters are also identified, further providing guidance towards achieving the target production outcomes at a lower cost.

2. Literature Review

Bayesian Belief Networks (BBNs) are known as probabilistic graphical models that represent variables and their conditional dependencies via directed acyclic graphs (DAGs), facilitating reasoning under uncertainty in complex systems (Barbrook-Johnson & Penn, 2022; Ben-Gal, 2007). Their graphical and modular structure provides a flexible framework for decision-making, diagnostics, and performance optimization in various industrial processes.

Particularly, in the domain of process modeling and quality optimization, Bayesian frameworks have demonstrated effectiveness in capturing dependencies between advanced manufacturing process inputs and output metrics. For example, in resin transfer molding (RTM) processes, BBNs have been used to model causal relations between tooling stiffness, injection parameters, and quality outcomes, supporting predictive control under uncertainty (Kempf & Hitzelberger, 2019). A similar knowledge-engineered BBN was introduced by Crawford et al., where process factors were discretized, structured, and scored based on domain expertise, supporting intuitive diagnostics and quality ranking (Crawford et al., 2020).

Recent advancements such as the dynamic-controlled Bayesian network (DCBN) have expanded the classical BBN capabilities by explicitly modeling dynamic responses to control inputs, demonstrating improved performance in tracking operational patterns in real time (Zheng et al., 2024). These techniques align with modern process control needs, particularly in systems where feedback and real-time inference are critical. Within thermoforming-specific studies, machine learning and neural networks have often been explored for process optimization. However, such methods typically require extensive datasets and may fail to capture causal relationships effectively. For instance, artificial neural networks (ANNs) have been used to predict process outcomes in composite sheet thermoforming (Tan & Nhat, 2022), yet these models are generally black-box in nature, and their performance can degrade in low-data regimes or when generalization is required across varied part geometries (Crawford et al., 2023).

In contrast, BBNs theoretically offer transparency and interpretability, particularly suitable when integrating physics-informed models and expert input. Several works have demonstrated the application of BBNs in other manufacturing domains such as drilling and machining, where parameter dependencies (e.g., feed rate, cutting speed, lubrication) are qualitatively modeled to support quality prediction and control (Kempf & Hitzelberger, 2019). In these models, critical to success is the accurate elicitation of conditional probabilities and meaningful discretization of continuous variables, a task often guided by engineering knowledge and empirical validation. Moreover, sensitivity analysis methods within BBNs enable the identification of dominant input variables that most affect output uncertainty, further guiding engineering decision-making around process optimization. Such analysis has been used in advanced curing processes (Crawford et al., 2023), as well as broader factory-level diagnostics frameworks (Ademujimi & Prabhu, 2024). For thermoforming, this is particularly relevant where thermal mismatches and material heterogeneities introduce complex interactions that traditional parametric models may overlook.

Earlier studies have also highlighted the role of BBNs in small-data environments. Ademujimi et al., introduced parameter-learning strategies that leverage domain constraints to overcome data limitations, a capability highly relevant for experimental and simulation-based manufacturing studies (Ademujimi & Prabhu, 2024). This addresses one of the main limitations of conventional statistical models in early-stage or small-batch production scenarios.

Despite these advancements, the present literature lacks studies that integrate multiple upstream heating and material parameters simultaneously into a unified probabilistic framework for thermoforming control. Most prior approaches either rely on physics-based simulations without interpretability, or use black-box models without causal reasoning. This gap underscores the novelty of the current study, which proposes a BBN framework that should not only capture the complex, nonlinear relationships between input parameters and quality indicators in thermoforming, but also offer an interactive decision-support tool for industry practitioners.

3. Methodology

In the present case study, a BBN was constructed to model the probabilistic dependencies among key thermal input parameters, intermediate process indicators, and final quality outcomes in a thermoforming process, as a case study. This process involves several interacting physical phenomena, and the BBN framework provides a coherent way to capture these interactions under uncertainty, especially in scenarios where empirical data is scarce or incomplete. The model incorporates controllable process inputs, such as heating power, sheet's material (thermal conductivity), and initial conditions (e.g., initial temperature), and links them to intermediate performance indicators (i.e., process stability, error, and settling time) to achieve the final product quality and energy consumption, as the outcomes. The structure of the proposed Bayesian Belief Network (BBN) was developed through a combination of engineering domain expertise and prior modeling insights, building on the earlier work by (Jalilvand et al., 2024), which focused primarily on optimizing heating parameters in the thermoforming process using reinforcement learning. In contrast, the present BBN framework integrates all key influencing factors **concurrently**, providing a more holistic representation that better reflects the complexity and interdependencies encountered in real-world thermoforming operations. More details are provided in Figure 1.

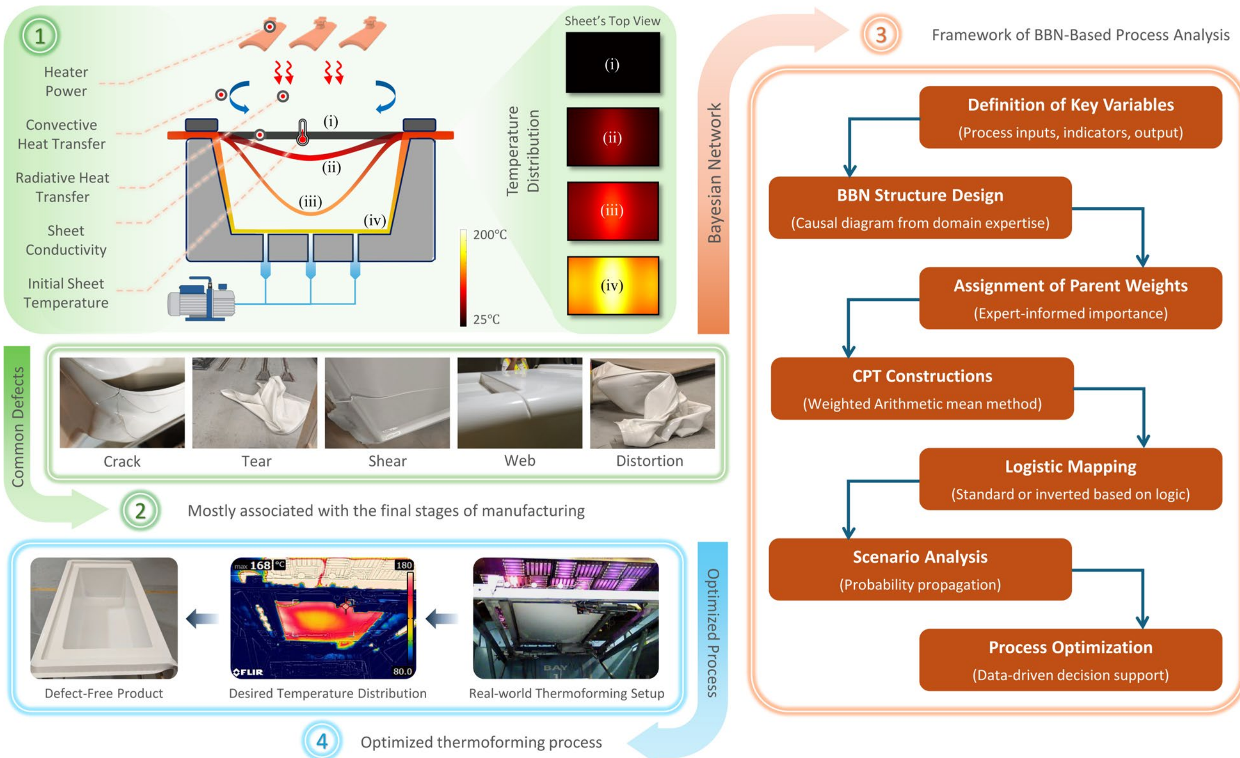

Figure 1. Overview of the proposed BBN-based framework for the thermoforming heating process optimization. The diagram integrates key process inputs such as heater power, convective and radiative heat transfer, sheet's thermal conductivity, and initial temperature (Section 1 in the image), the resulting temperature distribution and common defect types (Section 2), and the BBN modeling workflow (Section 3). The framework culminates in data-informed scenario analysis and decision-making to achieve an optimized, defect-free product (Section 4).

3.1 CPT Construction Using Weighted Arithmetic Mean

Each node in a BBN corresponds to a system variable (random variable), and directed edges denote causal relationships among them. The strength and nature of these relationships are captured via the so-called Conditional Probability Tables (CPTs), which define the probability of a node's state given the states of its parent nodes.

In contrast to data-driven BBNs, where CPTs are learned from large datasets, this study applies a structured **expert-guided** approach to CPT generation. Each CPT is built using a weighted arithmetic mean method, enabling interpretable and traceable probability calculations that reflect expert knowledge and engineering logic (Das, 2004). For a child node Y with parent nodes $X_1, X_2, \dots, X_n$, a weighted score is computed as follows:

$$Score = \sum_{i=1}^{n} w_i \, . \, x_i \tag{1}$$

where w_i represents the normalized importance weight of the parent x_i, and $x_i \in \{0,1\}$ denotes the numerical encoding of the state (e.g., "Low" = 0, "High" = 1). The weights are chosen based on both physical understanding and relative influence of each parent variable, ensuring the representation of plausible causality. The resulting score is mapped to a conditional probability using a sigmoid-shaped logistic function (Rijmen, 2008):

$$P(Y = high) = \frac{1}{1 + e^{-k(Score - 0.5)}} \tag{2}$$

This function provides a smooth transition between low and high probabilities, centered around the 0.5 score threshold. The parameter k controls the steepness of the curve, with higher values producing sharper transitions. In cases where higher scores represent worsening conditions, such as when parent nodes like "Error" or "Settling Time" are more detrimental in their "High" states, an inverted logistic function is applied, as indicated in Equation (3). Also, the comparison of the logistic and inverted logistic functions are reflected in Figure 2.

$$P(Y = high) = \frac{1}{1 + e^{k(Score-0.5)}} \tag{3}$$

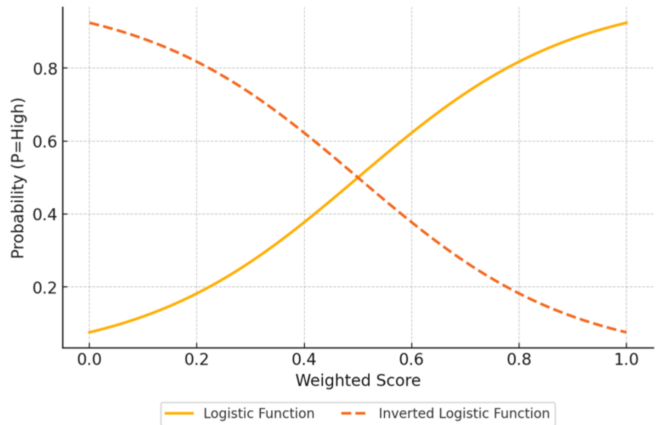

Figure 2. Standard logistic and inverted logistic functions used for score-to-probability mapping in CPT construction.

This mathematical inversion ensures that unfavorable parent states correspond to decreased probabilities of desirable outcomes, such as product compliance or process stability. This distinction is essential in accurately modeling scenarios where high parent values represent risk or inefficiency.

3.2 State Encoding and Logic-Driven Adjustments

The encoding of categorical states (i.e., "Low" and "High") into numerical values was carefully determined based on their physical implications. For instance, "High" initial temperature is favorable, and thus encoded as 1. On the contrary, "High" error or "High" convective heat transfer coefficient (HTC) may indicate problematic conditions, and although they are also numerically encoded as 1, their influence is adjusted using the inverted logistic transformation to maintain logical consistency in probability outcomes. This method allows the network to capture nuanced causal relationships without requiring additional hidden nodes or re-engineered structures. For example, the Process Stability is positively influenced by low error and settling time and negatively impacted by high power or HTC conditions.

3.3 BBN Structure and CPT Framework

The final BBN structure, illustrated in Figure 3, consisted of three hierarchical layers: input nodes, intermediate process nodes, and output nodes. The input layer includes controllable and environmental parameter, namely: Initial Temperature, Power, HTC, and Thermal Conductivity. Intermediate variables, including Error, Energy Use, Settling Time, and Process Stability, represent critical performance and operational indicators. These nodes aggregate and transform upstream information before influencing the output node: Product Quality. Each node's CPT was calculated based on the weighted arithmetic scheme, using combinations of binary states. An example CPT for the Error node is shown in Table 1. The CPTs for Energy Use, Settling Time, and Process Stability follow similar logic, with weights determined through iterative calibration against expert intuition. The categorical states "Low" and "High" used in Table 1 correspond to discretized physical ranges derived from the results obtained from a prior study (Jalilvand et al., 2025) empirical observations and expert-defined process limits. Specifically, for the current thermoforming setup under study:

- Initial Temperature: Low = 25 °C, High = 100 °C
- Power: Low = 0 W, High = 500 W
- Convective Heat Transfer Coefficient (HTC): Low = 5 $\frac{W}{m^2.K}$, High = 15 $\frac{W}{m^2.K}$
- Conductivity: Low = 0.04 $\frac{W}{m.K}$, High = 0.4 $\frac{W}{m.K}$

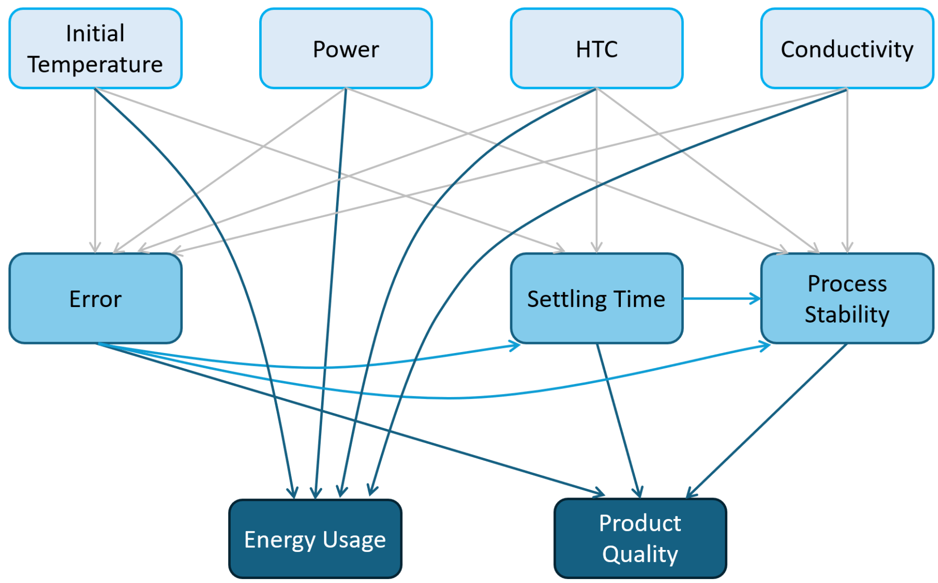

Figure 3. Structure of the Bayesian Belief Network (BBN) capturing causal dependencies across input, intermediate, and output variables. The parent (process control/input) nodes comprises the variables shown in the top row.

Table 1. Example the CPT for Error Node

Initial Temperature	Power	HTC	Conductivity	Weighted Score	P (Error = Low)	P (Error = High)
Low	Low	Low	Low	0	0.924	0.076
Low	Low	Low	High	0.15	0.852	0.148
Low	Low	High	Low	0.2	0.818	0.182
Low	Low	High	High	0.35	0.679	0.321
Low	High	Low	Low	0.4	0.622	0.378
Low	High	Low	High	0.55	0.438	0.562
Low	High	High	Low	0.6	0.378	0.622
Low	High	High	High	0.75	0.223	0.777
High	Low	Low	Low	0.25	0.777	0.223
High	Low	Low	High	0.4	0.622	0.378
High	Low	High	Low	0.45	0.562	0.438
High	Low	High	High	0.6	0.378	0.622
High	High	Low	Low	0.65	0.321	0.679
High	High	Low	High	0.8	0.182	0.818
High	High	High	Low	0.85	0.148	0.852
High	High	High	High	1	0.076	0.924

* *"Low" and "High" states correspond to the following ranges: Initial Temperature: 25 °C / 100 °C; Power: 0 W / 500 W; HTC: 5* $\frac{W}{m^2.K}$ */ 15* $\frac{W}{m^2.K}$*; Conductivity: 0.04 W/mK / 0.4 W/mK.*

These thresholds were selected based on prior thermal modeling and material property characterization, consistent with industry norms for acrylic-based composite thermoforming.

4. Results and Discussion

4.1 Baseline Inference

The initial baseline run of the developed BBN, i.e. with no evidence introduced, established a reference distribution across all variables. Under these default conditions, the output node *Product Quality* displays a balanced likelihood, with a 51.8% probability of being in the *High* state and 48.2% for *Low*. This suggests that the system, as modeled through expert-informed Conditional Probability Tables (CPTs), is positioned near a neutral outcome when no control action or external intervention is introduced. Among the intermediate performance indicators, *Error* shows a near-even split (53.3% Low, 46.7% High), and *Settling Time* (48.2% Low, 51.8% High) and *Process Stability* (54.0% High, 46.0% Low) follow a similar balanced trend. *Energy Use* remains almost evenly distributed (50.5% High, 49.5% Low), indicating no inherent bias toward efficient or inefficient operation in the default configuration. Notably, among the input variables, the prior probability was defined using the **reciprocal ranking approach**, which assigns weights based on expert-assessed desirability of each state. Specifically, the *High* state was ranked more favorable (rank = 1) and the *Low* state less favorable (rank = 2). The normalized probability for each state is computed using Equation (4):

$$w_i = \frac{1/r_j}{\sum_{a=1}^{n}(1/r_a)} \tag{4}$$

where r_j is the rank of the *j-th* state, and n is the number of states. Applying this to *Initial Temperature* yields a prior probability of 0.667 for *High* and 0.333 for *Low* (the rest of the input nodes' prior probability were calculated similarly). This reflects domain expert consensus that a higher initial sheet temperature contributes to faster achievement of target thermal profiles, supporting process stability and reducing defect risk. Figure 4 illustrates the full BBN model with these baseline state distributions, forming the basis for the scenario simulations and sensitivity analysis presented in the following sections.

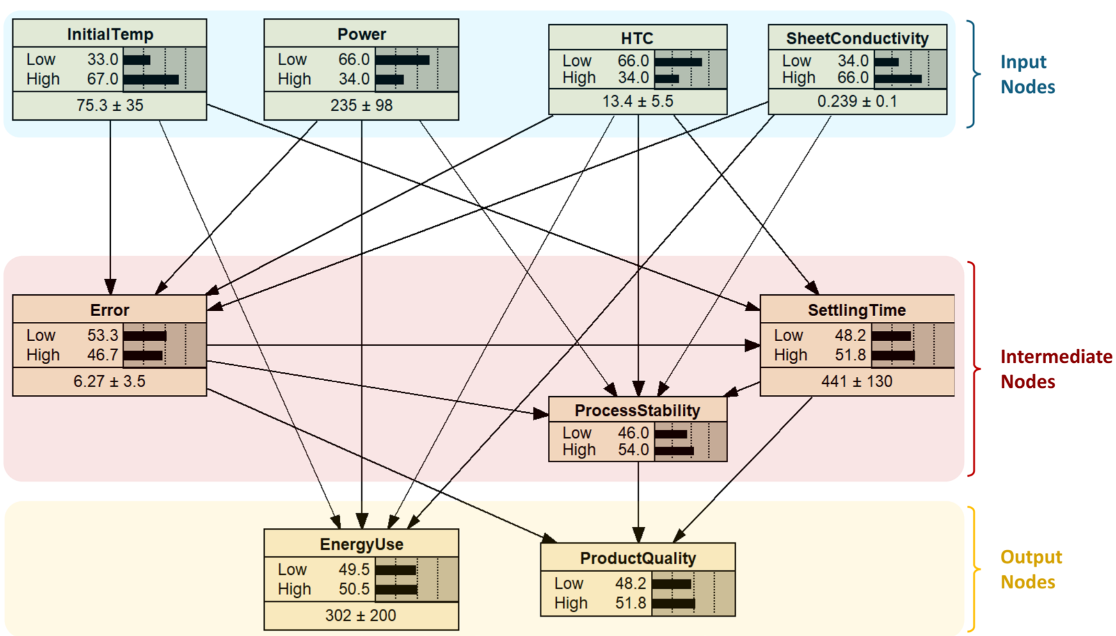

Figure 4. The BBN model marginal probability distribution under baseline conditions with no evidence set. The graph reflects the natural propagation of uncertainty across all nodes based on expert-informed CPTs.

4.2 Scenario Simulations: Product Quality

To identify the optimal input conditions required to achieve a desired outcome, a reverse inference strategy was employed. This backward reasoning approach allowed the model to infer the most probable configurations of upstream variables, both intermediate and input nodes, based on the desired output. To evaluate the contributing conditions that enable high product quality (desired outcome) **as case 1**, the **Product Quality** node was **set to High state** with full certainty. The resulting posterior distributions across the network reveal a shift in system behavior toward process configurations that are most likely to yield optimal outcomes. Notably, the three intermediate nodes, Error, Settling Time, and Process Stability, exhibit strong signals. Error shows a strong tendency toward the low state (82.5%), indicating that minimal deviation in thermal distribution or deformation is essential for product consistency. Settling Time is more likely to be Low (73.4%), pointing to quicker stabilization of process conditions. Process Stability sharply increases in the High state (80.4%), suggesting that consistency in operational parameters is a prerequisite for achieving high-quality outputs. Among the input parameters, several trends emerge that collectively reflect a controlled thermal strategy. Initial Temperature slightly favors the High state (56.8%), while Conductivity is also more likely to be High (61.8%). These two conditions together support a system that reaches the target thermal profile faster and more uniformly, reducing residual thermal gradients across the sheet. At the same time, both Power and HTC are substantially more likely to be Low (77.1% and 75.2%, respectively). This points to a reduced reliance on aggressive heating or cooling, which can otherwise introduce variability or surface defects due to uneven thermal exposure. Instead, the system appears to benefit from a more moderated thermal environment, one in which high initial temperature and enhanced conductivity enable efficient heating, while lower external inputs avoid thermal overshoots, further exemplifying the benefits of and an underlying need for a closed-loop control system. The resulting Energy Use profile aligns with this strategy, showing a dominant Low state (58.8%), reinforcing that high product quality can be achieved under energy-efficient operating modes. Taken together, these posterior shifts reflect an overall preference for a process that balances thermal input with material responsiveness, relying on intrinsic material properties like conductivity and initial sheet temperature to distribute heat evenly, rather than compensating through increased external input. The complete posterior state of the BBN under this constraint is visualized in Figure 5, serving as a graphical summary of the conditions most aligned with high product quality outcomes.

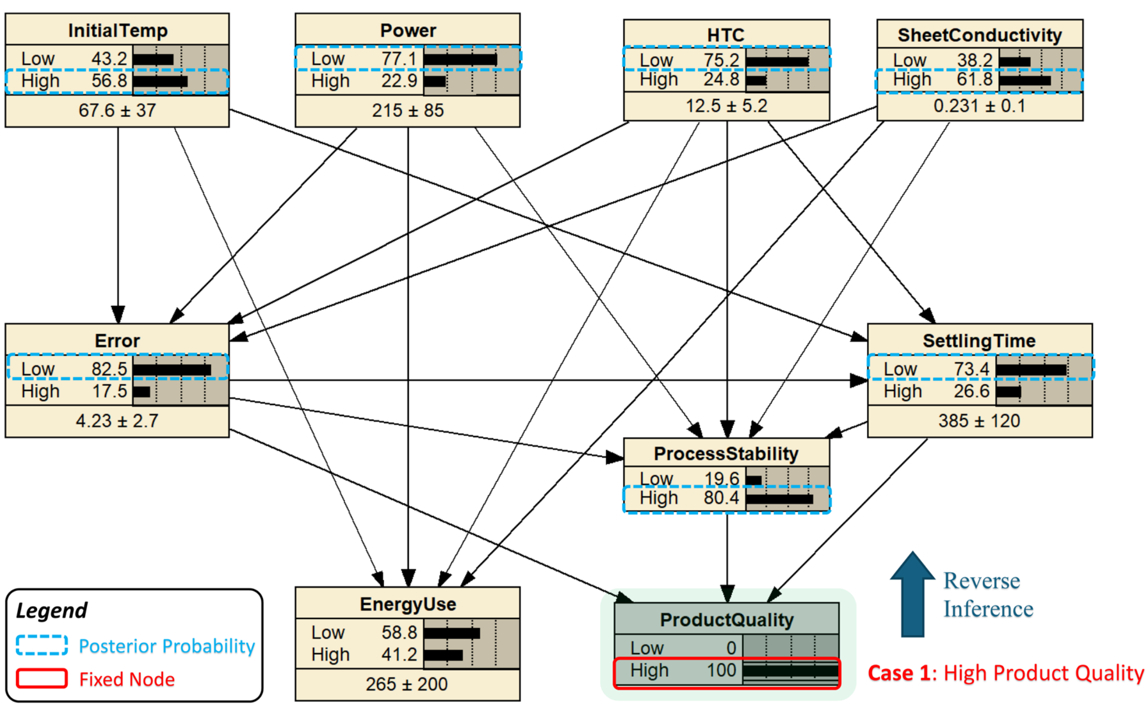

Figure 5. The BBN posterior inference when Product Quality was set to High (100%), identifying upstream conditions, such as, low power and HTC, and high conductivity and initial temperature, that contribute to optimal product outcomes (**Case 1**).

To explore energy-efficient operating conditions, **Case 2** applied reverse inference by **fixing the Energy Use** node to the **Low state**. Compared to Case 1 (High Product Quality), this scenario revealed a shift toward a thermally conservative strategy. Notably, Power and HTC showed strong posterior probabilities in the Low state (79.3% and 74.7%), indicating that reduced external inputs were critical for minimizing energy consumption. In contrast, Initial Temperature and Sheet Conductivity remained more balanced (51.5% and 59.5% High), suggesting that energy savings could be maintained without extreme material preconditioning. Intermediate nodes such as Error, Settling Time, and Process Stability showed slightly reduced performance relative to Case 1, but still favored their desirable states (65.0% low, 59.6% low, and 64.2% high, respectively), indicating that overall process control remained robust. Importantly, Product Quality also remained relatively high (61.5% High), confirming that energy reduction did not come at the cost of product integrity. These results highlighted the model's ability to expose meaningful trade-offs, showing that energy savings were achievable through reduced external heating and convection, provided that material responsiveness was maintained within a moderate range. The full posterior state under this constraint is shown in Figure 6.

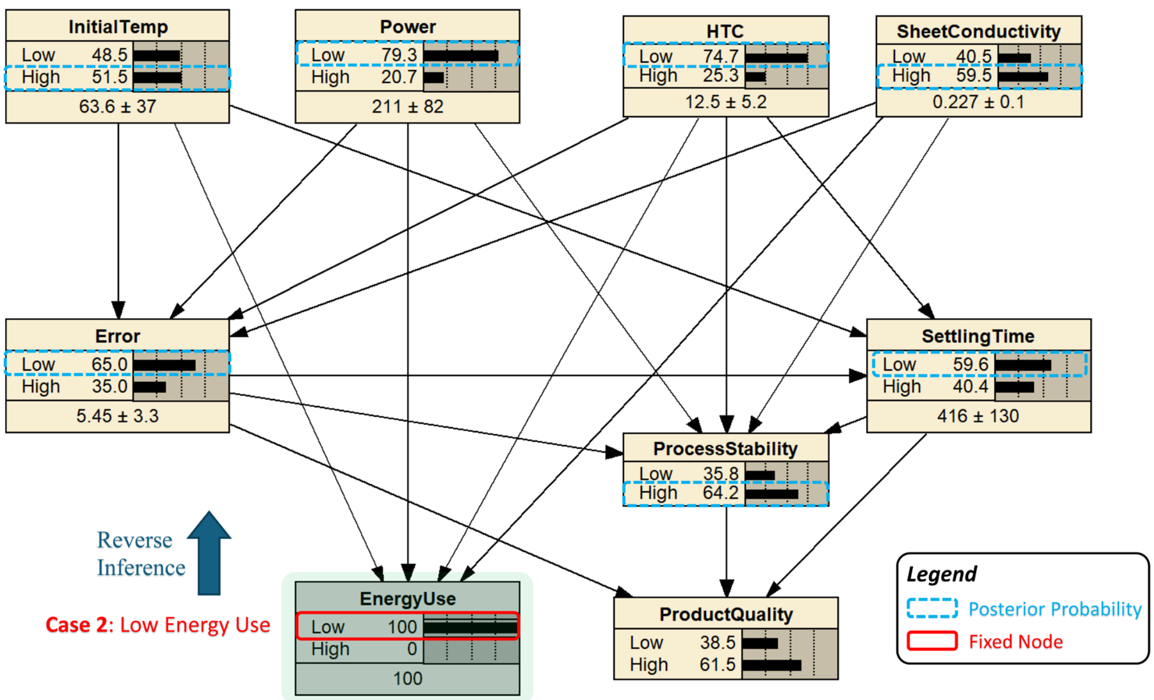

Figure 6. The BBN posterior inference when Energy Use was set to Low (100%), revealing upstream conditions that support energy-efficient operation without significantly compromising product quality (**Case 2**).

4.3 Sensitivity Analysis

A sensitivity analysis was conducted to further assess the influence of individual input parameters on key process outcomes by systematically fixing each input node to either its *High* or *Low* state. For each scenario, posterior probabilities of achieving favorable states were recorded for five critical indicators: *Low Error*, *Low Energy Use*, *Low Settling Time*, *High Process Stability*, and *High Product Quality*. The results are synthesized in Figure 7, which presents a heatmap that visually contrasts the performance of each input setting across all outcome metrics. Darker shades reflect a stronger likelihood of reaching the desirable state, making it easier to identify optimal and suboptimal configurations at a glance. Among all input nodes, **Initial Temperature** exerted the most pronounced effect. Setting it to *High* yielded the most favorable performance across every output: *Low Error* (68.9%), *Low Energy Use* (72.8%), *Low Settling Time* (77.5%), *High Product Quality* (67.8%), and *High Process Stability* (65.8%). These findings align well with thermoforming principles, where elevated preheating improves heat transfer rates, reduces the need for aggressive external inputs, and promotes faster thermal equilibrium, leading to more consistent part quality. In contrast, fixing *Initial Temperature* to *Low* significantly degraded system performance, with Product Quality falling to 34.9% and Error rising to 73.6%, reflecting the burden of compensatory heating and longer stabilization. **Power** showed similarly strong effects, particularly in its *Low* setting, where Product Quality improved to 60.5%, and both

Error and Process Stability reached more favorable levels. This suggests that once sufficient initial heating is achieved, limiting power prevents thermal overshoot and variability. Conversely, high power inputs mirrored the same degradation patterns observed with low Initial Temperature, confirming that excessive thermal input, if poorly synchronized with material conditions, leads to process instability. **HTC** exhibited moderate sensitivity. Reducing HTC improved Process Stability (64.1%) and Product Quality (59.0%), likely by minimizing surface temperature disruptions and enabling more uniform heating. In contrast, high HTC introduced greater likelihoods of prolonged Settling Time and suboptimal outcomes, possibly due to uneven convective heat transfer or premature cooling near the sheet surface. **Conductivity** displayed more nuanced behavior. While higher conductivity typically enhances internal thermal uniformity, in this case, setting it to *Low* resulted in better overall outcomes, particularly for Product Quality (58.2%) and Process Stability (62.9%). This may reflect an improved surface-to-core thermal gradient, allowing the outer layer to reach forming temperature before the core overheats. Such findings are consistent with recent literature in polymer processing, where moderated heat conduction has been shown to mitigate internal warping and enhance dimensional precision (Klein, 2009).

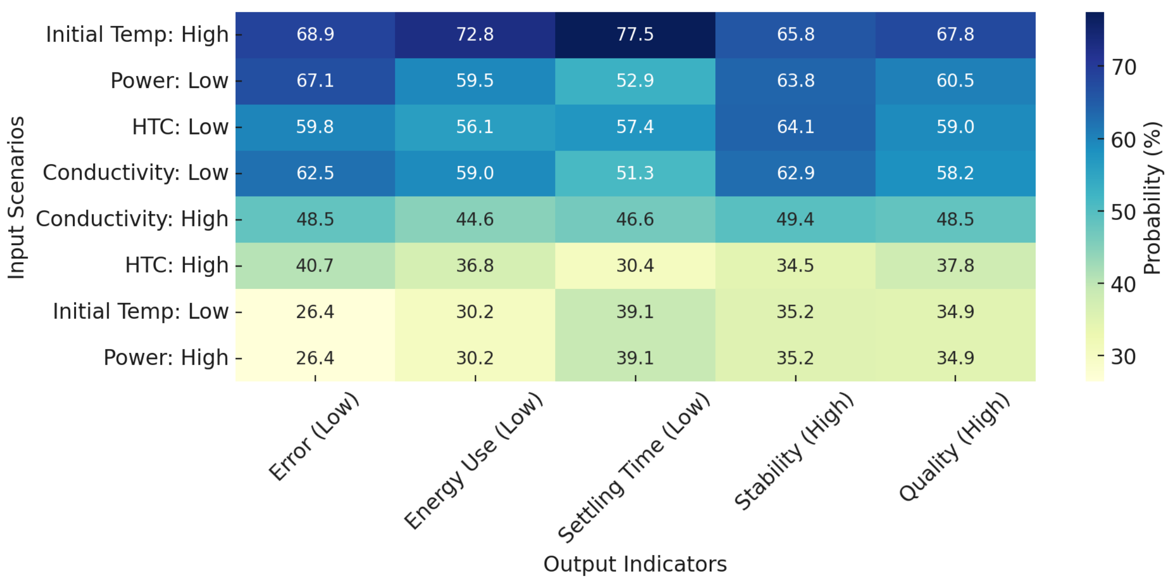

Figure 7. Heatmap showing the effects of individual process input (parent) nodes on key process outcomes. Each cell reflects the posterior probability (%) of a favorable state (e.g., low error, high stability) when the corresponding input is fixed to either High or Low.

To complement the scenario-based findings, a second layer of sensitivity analysis was performed using Netica's mutual information diagnostics. This approach quantified how much each variable, both inputs and intermediates, reduced uncertainty in the target node, Product Quality, when its state was known. As shown in Figure 8, the most influential contributors were *Error* (28.5%), *Process Stability* (23.0%), and *Settling Time* (20.7%), which together accounted for over 70% of the explained variation. Among the inputs, Power (4.3%) and Initial Temperature (3.7%) ranked highest, followed by HTC (2.9%) and Energy Use (2.7%), while Conductivity showed minimal direct influence (0.6%). These results not only highlight key process drivers but also help guide targeted investments. For instance, the high sensitivity to Error suggests that improving control systems, such as implementing closed-loop heater regulation or limiting uncontrolled airflow, could yield significant quality benefits. Likewise, stabilizing settling dynamics through predictive preheating or heater ramp control could further reduce process variability. Overall, this diagnostic reinforced the value of using BBNs to prioritize interventions that align with the most influential variables, enabling more effective and efficient thermoforming process improvements.

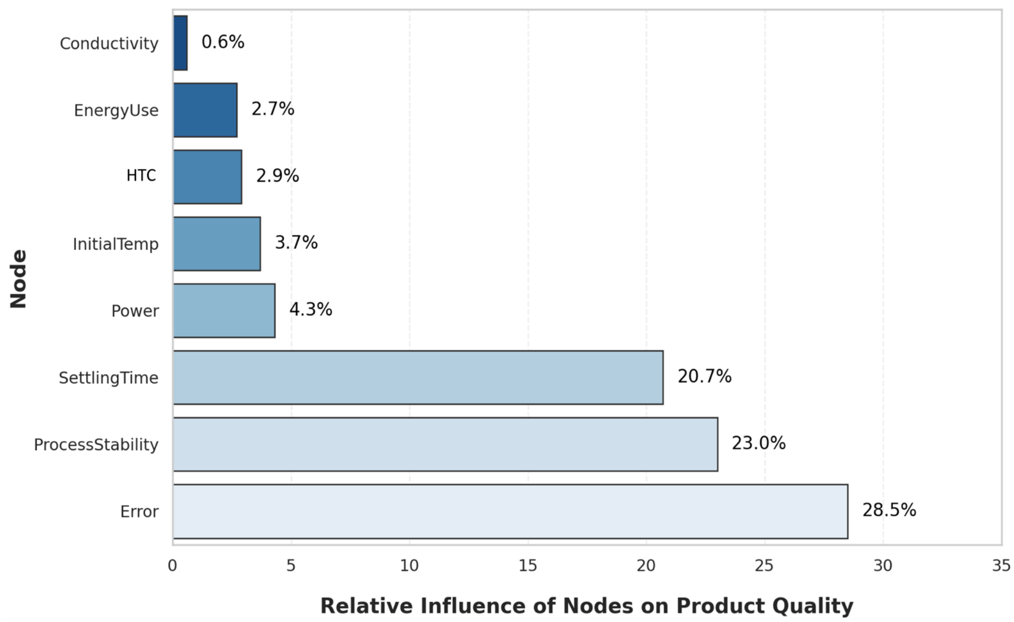

Figure 8. Sensitivity of Product Quality to findings at other nodes, ranked by mutual information (in %).

5. Conclusion

This study established a Bayesian Belief Network (BBN) framework tailored to the thermoforming process, addressing the concurrent effects of key input parameters on critical performance indicators. By integrating expert-informed CPTs with a weighted logistic mapping approach, the model effectively captured the probabilistic dependencies among variables such as initial temperature, heater power, HTC, and conductivity. The network enabled both forward and backward inference, allowing for scenario simulations and diagnostic queries on product quality outcomes. Sensitivity analysis revealed initial temperature and power as the most influential input nodes, while error, settling time, and process stability emerged as dominant intermediates in determining product quality. Diagnostic reasoning confirmed that high product quality correlates with low error rates, shorter settling times, and greater process stability, often achieved through moderate power input, controlled airflow (HTC), and favorable thermal conditions. These findings validate the model's capacity to support interpretable decision-making in data-limited thermoforming environments. Future work will focus on automating the CPT generation process through data-driven learning, and extending the framework to support decision networks with utility-based optimization, and possibly with integration of multicriteria material selection methods (Kasaei et al., 2014). This would enable dynamic control recommendations and further enhance adaptability in advanced manufacturing settings.

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Biographies

Iman Jalilvand is a Postdoctoral Research Fellow in Mechanical Engineering at The University of British Columbia (UBC), specializing in AI-driven process optimization, reinforcement learning (RL), and immersive extended reality (XR) development for manufacturing applications. His research integrates machine learning and XR to improve industrial training and optimize process control, with recent collaborations including KOHLER and Lululemon to name a few. Iman has received several academic honors, including UBC's Graduate Fellowship and the Silver Award at HCII 2023. Beyond his research, Iman has demonstrated leadership in both academia and industry. He serves as the founder of InnoXR, a startup delivering AI-enhanced XR training solutions, and has recently received the UBC Dean's Award for Knowledge Translation & Entrepreneurship.

Niloofar Akbarian-Saravi is a Postdoctoral Research Fellow in Mechanical Engineering, at The University of British Columbia (UBC), Canada. She expanded her knowledge in various fields including Bioproducts Supply Chain, Techno-Economic Analysis, Life Cycle Sustainability, and Multi-Criteria Decision Making (MCDM) and succeeded in publishing papers in prominent journals and proceedings. Her Ph.D. research at UBC is based on an industrial case (collaborated with Advanced Biocarbon 3D (ABC3D) Company) which incorporates economic, environmental, and social aspects in the entire supply chain along with employing MCDM and optimization approaches for selecting the best strategy. Recently, her research entitled "Analytic Network Process for Multicriteria Decision-Making in Sustainable Composites Supply Chain Management," won the 1st prize at 2024 Institute of Industrial and Systems Engineers (IISE) Sustainable Development Division Best Student Paper Competition, presented at the IISE Annual Conference in Montreal.

Bryn Crawford is a Research Engineer and Program Manager at The University of British Columbia's Materials and Manufacturing Research Institute (MMRI), responsible for establishing and coordinating circular economy-related R&D initiatives, particularly the PacifiCan-funded Accelerating Circular Economy (ACE) Platform. Bryn has experience working in both academic and industrial settings, executing over $20M of projects. The former largely in research centers collaborating with industry to solve technical barriers for introducing new products, materials, and processes, as well as root-cause analysis and continuous improvement projects.

Bhushan Gopaluni is a professor and associate head of Undergraduate Studies in the department of Chemical and Biological Engineering. Bhushan's primary research interests are in Process Modeling and Control, and is part of the Process Modeling and Control Lab. He is interested in developing and studying the properties of data-based models for a variety of chemical and biological systems. Bhushan is currently serving on the Executive Council for the North American Mixing Forum (NAMF), and is a member of the Association of Professional Engineers and Geoscientists and British Columbia (APEGBC), Canadian Society for Chemical Engineering (CSChE) and Pulp and Paper Technical Association of Canada (PAPTAC).

Abbas S. Milani is a Professor of Mechanical Engineering, and Tier 1 Principal's Research Chair in Sustainable and Smart Manufacturing at the University of British Columbia (UBC), Canada. He is a leading expert in the field of composites and bio-composites manufacturing with applications of Industry 5.0 under multiple-criteria decision-making/optimization environments. He is a highly community-engaged researcher who excels in bringing together people and ideas. He is the founding Director of UBC Materials and Manufacturing Research Institute, Lead of the Canadian-International Biocomposites Research Network, and Technical Director of the Composites Research Network (CRN) in Canada. He was inducted into the Royal Society of Canada in 2020.