Proceedings of the 8th European Conference on Industrial Engineering and Operations Management
Paris, France, July 2-4, 2025

Proceedings of the International Conference on Industrial Engineering and Operations Management

Publisher: IEOM Society International, USA DOI: 10.46254/EU08.20250382
Published: July 2, 2025

Analysis of the Performance and the Behavior of Bass Diffusion Model in Forecasting Demand for Fashion Goods: A Case Study

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Abstract

Forecasting sales is a key factor for efficient operational decisions in replenishment planning and inventory control in almost all supply chains. For demand forecasting, the model introduced by Bass (1969) has been used in several studies with different type of products in literature. In this study, we apply Bass diffusion model on a large-scale real-world sales dataset from the fashion industry. In estimating the parameters of the model, we use three common parameter estimation methods from the literature and introduce a new method called randomized line search. We analyze the forecasting performance of these methods in estimating the demand in a sale season using the entire dataset involving several different product categories. Based on the mean absolute percent error criterion, we check how good Bass diffusion model works and the performance of different methods for finding the parameters of the model, across 409 different products using a real data set from an apparel retail chain with 150 stores.

Keywords
Sales forecasting, Fashion products, Bass diffusion model

1. Introduction
Modeling and forecasting sales or demand is a critical managerial function and forecast accuracy is directly related to the financial performance of the firms, especially for new products or seasonal fashion products. A seminal approach on modeling the customer demand for a new product is given in Bass (1969) as a diffusion model. We will call this model Bass diffusion model (BDM). BDM takes customers into consideration as a network of individuals who influence each other. The success of BDM in modeling sales data has been shown in Bass (1969) and several follow-up studies that we list in the literature section.

Fashion industry can be defined as the work of producing and selling clothes, and it is a multi-billion-dollar global enterprise. In this study, we analyze the performance of BDM in forecasting future demand using a real data set that we obtained from a company which is in apparel and fashion goods business. The company both produces and sells its products through more than 150 retail stores in Turkey. The case study includes 409 different products.

The competition among apparel companies is severe, and they are all keen to reduce the costs related to their operations. The competitive environment of the industry requires a significant amount of effort in many aspects in operations management. One of these aspects is the forecasting demand or sales. Fundamentally, sales or demand forecasting can be defined as the estimation of the future sales of product based on historical data, market trend, product characteristics or other relevant factors (Du et al. (2015)). Accurate prediction of demand is the key for successful supply chain management in general and, in particular, for fashion products in the apparel industry with significantly changing demand patterns from a sales season to the next. Considering the severe competitive environment in the apparel industry, we can say that accurate sales forecasting is gaining more and more importance in the industry due to its ability to provide a basis for improving operational decisions, reducing overproduction, lost sales or need for markdowns (Nenni et al. (2013)).

Our aim is to analyze the performance of BDM in predicting sales curve for fashion garments using both full datasets based on the mean absolute percent error criterion. With the data on complete sales, we build a model for each product to see if BDM is capable to represent total seasonal demand. In estimating the parameters of BDM, we use both methods from literature and also suggested a new method. The remainder of study is organized as follows; the next section gives the relevant literature. Section 3 involves th5 explanation of the methodology. Section 4 describes the application of the methodology on the case data, and section 5 gives our conclusions and future research directions.

2. Literature Review

2.1 Bass Diffusion Model

After the introduction of the BDM by Bass in 1969, many related works in the literature followed this seminal study. We can classify these follow up studies into two main categories; the studies that extend the structure suggested in BDM and the work about developing different parameter estimation methods for BDM. We would like to briefly introduce BDM model here for the sake of completeness. BDM model mainly depends on the probability of adoption of an individual for a new product at time t. The hazard rate, probability distribution function, probability density function, and the amount of sales at time t, $S(t)$, in BDM are given in Equations 1, 2, 3, and 4, respectively, as follows;

$$\frac{f(t)}{1-F(t)} = p + qF(t) \tag{1}$$

$$F(t) = \frac{1-(p+q)t}{\left(1+\frac{q}{p}\right)e^{-(p+q)t}} \tag{2}$$

$$f(t) = \frac{\frac{(p+q)^2}{p}\left(e^{-(p+q)t}\right)}{1+\frac{q}{p}e^{(-(p+q)t)^2}} \tag{3}$$

$$S(t) = m \times f(t) \tag{4}$$

The parameters p and q in Equation 2 are innovation and imitation coefficients, respectively. Innovation coefficient represents the proportion of people who would adopt a new product, without the influence of other people. Imitation coefficient is the proportion of people who adopt a new product after seeing other adopters of the new product. For further details we refer the reader to Bass, 1969. There are several attempts to incorporate different factors into BDM in order to enhance the flexibility and the performance of the model. The main idea behind these extensions is incorporating the variables related to the products, the target market, or the environment. In the literature, price variable is seen as one the most important explanatory variables, and considerable number of studies conducted to integrate the price variable into BDM. Robinson and Lakhani (1975) is an early example of a structural extension of BDM, and it offers inclusion of price variable to the model (Robinson and Lakhani (1975)). Other examples of studies that incorporate price issue are Robinson & Lakhani (1975) Mahajan & Peterson (1978), Bass (1980), Bass & Bultez (1982). Another study, Tanner (1978), used gross domestic product per capita as an additional explanatory variable to BDM model. There are extensions of BDM based on the modification of the hazard rate as well. Robinson and Lakhani modified hazard function as follows.

$$\frac{f(t)}{1-F(T)} = \left(\beta_0 + \beta_1 F(t)\right)e^{\beta_2 P(t)} \tag{5}$$

Where $P(t)$ represents the price in the model at time t. With this addition, they aim to investigate the pricing effect on the model. It was shown in the literature that the extensions based on addition of explanatory variables, such as price and advertisement, make the model more accurate, although the number of parameters to be estimated increases. For example, Bass et al. (1994) suggest a "Generalized Bass Model" (GBM), which incorporates a new variable named "current marketing effort" as a representation of advertisement and price variables. The hazard function with the inclusion of the current marketing effort variable is given as:

$$\frac{f(t)}{1-F(T)} = \left(\beta_0 + \beta_1 F(t)\right)x(t) \tag{6}$$

Where $x(t)$ represents mapping function for current marketing effort. If x(t) is taken as equal to one across periods, GBM reduces to BDM. The authors also include a comprehensive review of the literature on BDM until 1994. In our data, price does not change significantly from week to the next, we used BDM rather than GBM. Additionally, it is shown that if there is considerable change in prices and advertising effort during sales periods, GBM gives better fit than BDM.

The success of BDM in reflecting the sales curve is shown by several research works over the years. However, this success highly depends on the accurate estimation of the parameters of BDM. Due to the dependency of success of BDM to the parameters, several perspectives are developed to estimate them. We will go over four different types of parameter estimation techniques; the first one is the ordinary least squares parameter estimation (OLS) technique, which is used in Bass (1969) with the introduction of the original model. In OLS method, the three parameters are calculated by equating the BDM to a regression model of the sales data. It is found that OLS gives acceptable parameter estimations. Subsequently, as a second method, Schmittlien & Mahajan (1978) utilize maximum likelihood parameter estimation (MLE) method for parameter estimation(Schmittlien and Mahajan (1982)). They show that in certain data sets MLE is superior to OLS. Srinavasan and Mason (1986) estimates the parameter of BDM with a non-linear least squares parameter estimation method (NLS) as a third method. There are several studies in the literature on comparing parameter estimation methods (e.g., Hong et al. (2016) and Meade & Islam (2006)). In general, the researchers show that both MLE and NLS are superior to OLS. Based on the studies in the literature, although it may seem NLS more successful in terms of estimation errors, there is no consensus that NLS is better than MLE. Another parameter estimation method is based of genetic algorithm (GA) and it is suggested by Venkatesan et al. (2004). This study criticizes former parameter estimation methods in different aspects, such as less chance of stacking at a local minimum and proposes the use of GA to estimate the parameters. They show that GA is better than NLS in their work, although not in a significant way. The disadvantages of GA method are both that it requires long computation time, and the success of GA is directly dependent on the parameter values used in GA, such as mutation rate, cross over rate, population size, etc.

There are more recent studies on parameter estimation as well. For example, Hong et al. (2016) showed the integrated use of OLS and NLS. Another more recent study claims that NLS, MLE and OLS are not good for data sets without a peak demand period, and they use a Bayesian approach in parameter estimation Hassan and Montoya-Blandon (2019). BDM is utilized to describe the sales curves for variety of products in the literature. These products mainly consist of technological products, such as color TV, compact disc (CD), steam iron, all of which involve innovations. The review conducted by Meade and Islam (2006) emphasizes that for a good fit of BDM, the product should be of adoptable type rather than consumable. This precondition ensures that repetitive purchases by an individual will not occur. The type of products we study in this work is of adoptable type and we can expect a good fit using BDM.

2.2 Sales Forecasting in Fashion Industry

The need for forecasting demand accurately is very important and crucial in the fashion industry. Considering characteristics of fashion products, such as short life cycles and long lead times, it is critical to achieve accurate forecasts for a firm from a competitiveness point of view (Nenni et al. (2013)). Wrong forecasts directly lead to cost in the form of inventory holding, lost sales, and increased portion of products sold with markdown prices at the end of the sale season. Nevertheless, predicting the sales curve for fashion products is usually difficult due to the high variability of demand. The demand pattern of fashion products can be classified as intermittent and erratic. It is very common to see periods with very low or zero demand and then sudden peaks in demand in the following periods. An early traditional statistical method suggested for forecasting demand with high variability is Croston's method (1972) which is developed for forecasting intermittent demand. Another study that considers the sale process as a non-homogeneous Poisson process and accepts sales at time t as an event in the Poisson process (Wang et al. (2006)). There are also contemporary approaches in forecasting fashion products as well, such as artificial neural networks (ANN) (Gutierrez et al. (2008)). Due to the flexibility of ANN to reflect data with lumpy and erratic pattern, as it is the case in fashion product sales, few variants of ANNs were used in the literature in forecasting demand of fashion products. Extreme learning machine (ELM) can be considered as one of ANN approach (Sun et al. (2008)). Wong and Guo (2010) utilizes ELM and tuning ELM with improved harmony search method. A comparison between ARIMA and suggested ANN is also included in this study, and they show the superiority of the proposed ANN model (Wong and Guo (2010)). Du et al. (2015) also use ANN with different extensions, in the literature it seems that there is consensus on using ANN to describe demand for fashion products. Most of the research embraces ANN as the proper structure and combines it with different optimization techniques, such as multi-objective optimization algorithms and harmony search algorithms. These algorithms are utilized to find parameters of ANN such as, the number of hidden layers, learning rate, the number of hidden nodes, etc.

To the best of our knowledge, there is no comprehensive study on the applicability of BDM on fashion products. The only work related with Bass diffusion model and fashion products is, Namin et al. 2017 which mainly proposes markdown policies for fashion products, and it only assumes demand pattern follows BDM. It neither analyzes the performance of BDM nor studies how the parameters of the model change depending on the amount of data, as we do in this study.

3. Methodology

In this section we describe both three parameter estimation methods from literature and a new method we suggest in this study, called randomized line search. Before we can use the data, we eliminate the end of season abnormalities in data, which we describe in the following section.

3.1 Data Truncation Process

There are some abnormalities in the data caused by the end of season promotions and outlet sales. BDM is expected to represent the data without these end-of-season abnormalities which are due to mostly significant markdowns. Therefore, we decided to truncate the data to exclude the abnormal part before fitting a BDM as it has been applied in the literature. We truncate the data from a point which is calculated based on both the volume of the sales and the number of available data points. The first truncation point is determined as the period at which is not less than or equal to %90 of available data periods. The second truncation point is determined as the period by which %90 of total sales occur. We choose the maximum of these two points as the truncation period. An example truncation process can be seen in Figure 1.

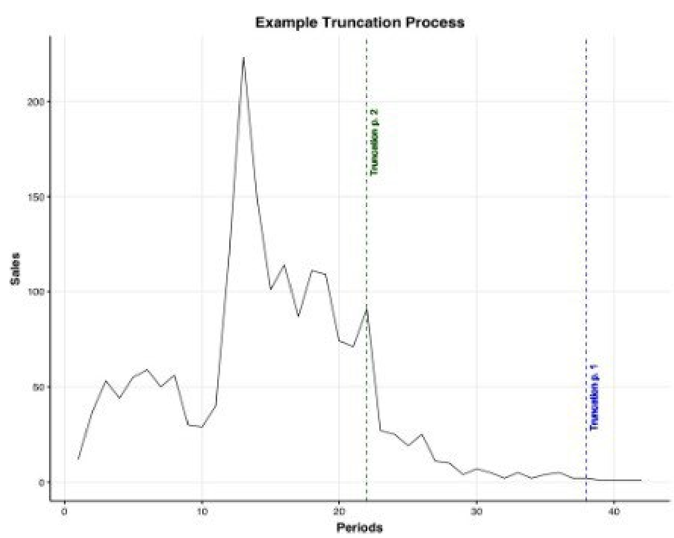

Figure 1. Truncation points in seasonal data

3.2 Parameter Estimation Methods in Literature

For extensive analysis of the Bass model, we used three main proposed parameter estimation techniques in the literature, which are non-linear least squares estimation (NLS), maximum likelihood estimation (MLE) and ordinary least squares estimation (OLS). In addition to these methods from the literature, we also propose a new method that we call randomized line search algorithm (RLS). In this section, we briefly describe the three methods from the literature and also the novel approach we suggest. In OLS, parameters are estimated by equating the BDM model to a regression model, which has cumulative sales and cumulative sales squared as independent variables.

$$S_t = \beta_1 + \beta_2 Y_{t-1} - \beta_3 Y_{t-1}^2 \tag{7}$$

Equation 7 shows the linear regression model where Y_t represents cumulative sales until the period t, and S_t stands for demand in expected in period t. This expected value is equated to.

$$S_t = pm + (q-p)Y_{t-1} - \frac{q}{m}Y_{t-1}^2 \tag{8}$$

which is an expanded version of BDM, given in Equation 7 (Bass, 1969) From this relation, we can set; $\beta_1 = pm$, $\beta_2 = q - p$ and $\beta_3 = \frac{q}{m}$ and obtain the parameter estimations using the following three equations.

$$\hat{p} = \frac{-\beta_2 + \sqrt{\beta_2^2 - 4\beta_1\beta_3}}{2} \tag{9}$$

$$\hat{q} = \frac{\beta_2 + \sqrt{\beta_2^2 - 4\beta_1\beta_3}}{2} \tag{10}$$

$$\widehat{m} = \frac{-\beta_2 + \sqrt{\beta_2^2 - 4\beta_1\beta_3}}{2\beta_3} \tag{11}$$

In our study QR-Decomposition [23] is utilized to minimize the objective sum of the squared difference in fitting the regression model. In NLS method, we estimate the parameters by directly minimizing the sum of the squared differences between the actual and the demand using BDM model. The demand in a period t_i is given as following.

$$S(t_i) = m[F(t_i) - F(t_{i-1})] \tag{12}$$

If we let $y(t_i)$ represent the observed demand in period t_i, in NLS we minimize $SSE = \sum_{i\in\{1..T\}}[y(t_i) - S(t_i)]^2$. We obtained the minimizing values of the parameters by applying Gauss-Newton algorithm [24]. This algorithm requires starting values. We used the parameter estimations obtained from MLE approach as the starting points in NLS. But we observe that there is no considerable effect of different starting points. Regarding MLE approach, Schmittlien and Mahajan (1982) express adoption by time t (i.e $F(t)$) as in [11].

$$F(t) = \frac{c(1-e^{-bt})}{1+ae^{-bt}} \tag{13}$$

Using Equation 13 likelihood function is written as in Equation 14.

$$L\big(a,b,c,y(t_i)\big) = [1 - F(t_{t-1})]^{y(T)} \prod_{i=1}^{T-1} [F(t_i) - F(t_{i-1})]^{y(t_i)} \tag{14}$$

Equation 14 shows the likelihood function that provides the base to the MLE. It represents the likelihood of observing the demand that is observed in a season. Here as period T, remaining time after the season end is assumed. Thus, for $y(t_i)$ up to period T we plug in the observed demand while for $y(t_T)$ we use total realized sales up to time. On the right-hand side of Equation 14, the first component represents the likelihood of observing the remaining demand after the season while the second component is the likelihood of the observed demands in all the periods within the season. Further details of how likelihood function for BDM is generated are given in Schmittlien and Mahajan (1982). Since there is no information about sales at period T, this value is obtained using estimated the number of eventual adopters. Likelihood function also involves parameters as a and b, where $a = q/p$, and $b = p + q$. The parameter c represents the probability of eventual adoption, and this parameter relaxes the assumption of BDM that all potential adopters eventually adopt the product. The goal in the MLE is to maximize the likelihood function. In our study, we did this maximization process with Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm (Nash 2017). Since, the algorithm works for minimization case, and we minimize the -$log(L(a,b,c,x_i))$.

3.2 Randomized Line Search (RLS)

As a fourth method, we suggest a randomized line search in order to minimize SSE. We search for the best triplet of parameters (p, q, m) in RLS. The pseudo code of the Algorithm for RLS is given in Figure 2. The algorithm first creates a set of N triplets of parameters as the set of alternative starting points. Starting with each of these N triplets, we reach to N final solutions. p and q are searched in the range [0, 1] and m is searched in a range from half the total demand observed in the season to twice the total demand. We also use the same ranges in randomly determining N starting triplets of values.

```
Data: Weekly Sales
Result: Estimation of p, q and m
Initialization;
Generate (p, q, m) triplets randomly.
forall Starting Points do
    attempt ← 1;
    fitness1 ← calcFitness(p,q,m);
    while n < iterationLimit & attempt < k do
        while p != ChangeP(p, fitness1) do
            p' ← ChangeP(p, fitness1);
            p ← p';
            fitness1 ← calcFitness(p, q, m);
        end
        while q != ChangeQ(q, fitness1) do
            q' ← ChangeQ(q, fitness1);
            q ← q';
            fitness1 ← calcFitness(p, q, m);
        end
        while m != ChangeM(m,fitness1) do
            m' ← ChangeM(m, fitness1);
            m ← m';
            fitness1 ← calcFitness(p, q, m);
        end
        n ← n + 1;
        fitness2 ← calcFitness(p, q, m);
        solution pool ← fitness2 ∪ solution pool
        if fitness1 = attempt2 then
            attempt ← attempt + 1;
        end
    end
    if fitness2 ≤ the minimum fitness value in the solution pool then
        p* ← p;
        q* ← q;
        m* ← m;
    end
end
return p*, q* and m*;
```

Figure 2. Algorithm for Randomized Line Search

In the method which presented as a pseudo code above, since the values of p and q should be between 0 and 1 we develop a line search method that searches feasible space of BDM parameters. Algorithm moves in the direction of improvement along the line with randomized step size. Direction of improvement is determined by adding or subtracting random step size from the current value of the parameter. Then with the assistance of $calcFitness$ function we evaluate the SSE values for both parameter estimations. If achieving smaller SSE value is possible, the functions ($changeP$, $ChangeQ$ and $ChangeM$) returns p', q' or m' values, respectively. If it is not possible the functions return the current values of parameters. The pseudo code of function $changeP$ is given below, the function called step() is used for generating random step sizes for parameters.

When the point with no available improving move is reached, algorithm does not stop. It increases the number of attempts variable by one. If the number of attempts exceeds the attempts limit, which is denoted by k in the pseudo code, algorithm exits the loop and checks whether reached p, q and m values give better fit than the parameter estimations in the pool. If obtained values are the best, the $p*$,$q*$ and $m*$ values are updated with the found values. Parameter estimations reached from each starting point are added to the solution pool, which will eventually have N members. Finally, algorithm returns $p^\star$, $q*$, and $m*$ values which have the smallest SSE value.

4. Analysis of the Case Data

After preprocessing, the data consists of weekly sales of each product. We had weekly sales data for 409 different products. We divided the data into eleven subgroups according to the family of the products as given in Table 1. We determined these subgroups considering both the product types and expert opinions.

Table 1. Distribution of Products into Categories

Categories	**Number of Products**
Category 1	23
Category 2	41
Category 3	39
Category 4	22
Category 5	14
Category 6	44
Category 7	28
Category 8	54
Category 9	24
Category 10	43
Category 11	77
Total	409

In this section we want to check the representative power of the BDM in our case data. We fit a Bass diffusion model to the data points for each product and construct the BDM which is used to draw Bass curves. Using the fitted BDM, we estimated the total demand for each week in the season and for each product. Then we calculated mean absolute percent error (MAPE) of the model in estimating remaining demand in the season. Remaining demand is calculated from periods (i.e., 1, 2, 3..., $T-1$) until the end of the truncation point (i.e., T) that effectively represents the end of sale season. Then, we obtained the MAPE values by comparing the actual remaining demand and the remaining demand found by fitted BDM. Table 2 shows the average MAPE values for each category of the products, by using four different parameter estimation techniques. Bold values represent the best solution for each category. From the table, we can say that NLS seems to be superior to other techniques. However, in some product categories MLE gives better results. It is also worth noting that the method we suggested, RLS, performs quite comparable with MLE and NLS, and is better than OLS method, in terms of MAPE values. Thus, we can consider our method as an effective method.

Table 2. MAPE values for different methods

Category	**OLS**	**NLS**	**MLE**	**RLS**
Category 1	6.87	**6.44**	6.71	7.23
Category 2	8.24	7.31	**7.11**	7.84
Category 3	8.96	**6.49**	7.18	7.66
Category 4	12.74	**7.54**	8.52	8.33
Category 5	7.67	7.72	**7.2**	8.66
Category 6	7.84	**6.15**	6.39	6.87
Category 7	8.27	**6.89**	7.05	8.13
Category 8	11.46	**6.93**	8.2	8.18
Category 9	9.2	**7.07**	7.63	7.85
Category 10	9.02	**6.64**	7.22	7.91
Category 11	7.1	**5.98**	6.62	6.63

Mean MAPE	8.85	**6.83**	7.26	7.75
Std of MAPE	1.79	**0.55**	0.64	0.62

An important observation is that the MAPE values on the table are less than 10 percent in general, which represents a satisfying estimation error especially for fashion products. Thus, we can say that, overall, BDM seems to be a suitable choice to represent the demand trajectory of the products in our case data and in fashion products generally speaking. Furthermore, the analysis made on the residuals shows that the residuals are normally distributed with mean of 0 and a constant variance, based on the results obtained using NLS technique. To check normality of residuals, we used Shapiro-Wilk and Kolmogorov-Smirnov tests, p-values gained from these tests show that there is no strong evidence against normality of residuals. Thus, BDM model seems to be an unbiased estimation model in our case data. Also, to examine whether residuals are auto-correlated or not, we utilized Durbin-Watson test with the null hypothesis of residuals are not auto-correlated. As a consequence, we found out p-values larger than 0.05. Therefore, we can say that the residuals are not auto-correlated (Durbin and Watson (1951)). Figure 3 shows an example of forecast curve based on BDM and the actual sales quantity by weeks. We can observe the erratic pattern of the actual demand curve.

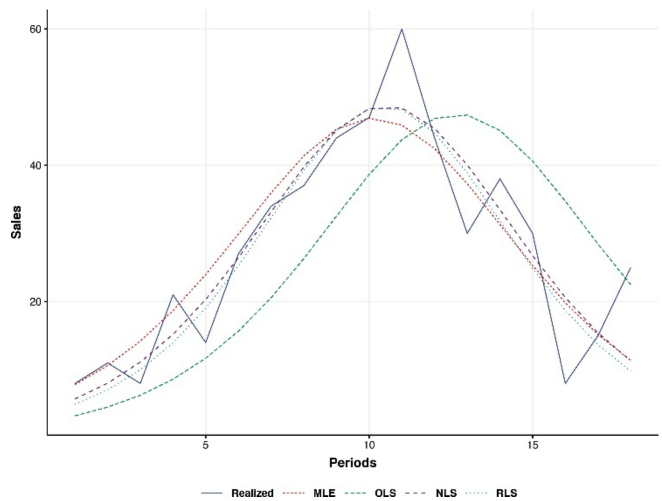

Figure 3. Sales and fitted bass curves

The erratic pattern of the demand curve is of secondary importance since what is important is to forecast the remaining demand in a season and therefore, we calculate the performance of BDM based on the remaining demand in a season. We believe that the prediction of remaining demand in a season is an important insight into operational decisions, such as the number and the amount of reorders in the middle of the sale season.

5. Conclusion

Our primary aim was to analyze whether the demand of fashion products, as adoptable products, can be subject of diffusion process or not and if they can be modeled accurately enough using BDM model. We found out that it is possible to represent the sales curve of a fashion product by using only three parameters (i.e., p, q, m) within BDM. As one can see in Table 2, we can estimate the demand in a season with approximately 6-7% MAPE values in general using BDM, which is a very promising MAPE. Another conclusion is that our suggested method for parameter estimation outperforms OLS method, and it is quite comparable to two traditional estimation methods, namely NLS and MLE.

As for the future research we can mention three aspects that can be considered as complementary to our study as follows. Since the company operates in different countries, or regions, each area may react differently to the new products. Therefore, we can develop BDM model based on geographically separated data. Another field of future research is defining similarity between products regarding their BDMs and sales trajectories. This issue can be handled by clustering techniques, and the similarity between products can be identified statistically. Furthermore, a dynamic forecasting method can be developed by using offline information obtained by BDMs of past products and online information gained from partial observation of the product sales. Finally, since BDM is found to be able to represent the sales curve of fashion goods, the parameters of fitted BDM models can also be utilized in customer segmentation applications.

Acknowledgment

We acknowledge and appreciate the support of Qatar University

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Biographies

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