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# A Comparative Study of LQR and FHLQT for Ground Vehicle Lane Change Maneuvers

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## Abstract

This study presents a comparative analysis between two optimal feedback control laws—Linear Quadratic Regulator (LQR) and Finite-Horizon Linear Quadratic Tracking (FHLQT)—for autonomous ground vehicle lane-change maneuvers. While the conventional LQR employs a fixed gain matrix optimized over an infinite time horizon, the proposed FHLQT method utilizes a time-varying control gain derived from solving the Differential Riccati Equation (DRE) backward in time. The vehicle model, based on a nonlinear bicycle dynamics framework, is linearized for controller synthesis, and simulations are performed under varying operational conditions. Results demonstrate that the FHLQT controller provides superior tracking performance, faster transient response, and improved robustness against parameter variations when compared to the fixed-gain LQR. The study concludes that FHLQT offers a more efficient control strategy for short-duration dynamic maneuvers such as step lane changes, despite its higher computational cost.

## Keywords
Optimal Control; LQR; Lane-Change Maneuver; Ground Vehicle Dynamics; Autonomous Vehicle Steering; FHLQT

## 1. Introduction
The Linear Quadratic Regulator (LQR) remains one of the most widely applied optimal state-feedback control techniques in modern control theory. Originating in the latter half of the twentieth century, it provides a systematic way to minimize a quadratic performance index that balances state deviation and control effort. The LQR framework is particularly valued for its guaranteed stability margins—such as infinite gain and a minimum phase margin of approximately 60° in single-input systems—and its capability to produce smooth and efficient control actions across diverse engineering domains.

In the context of vehicle dynamics and path tracking, LQR formulations have been extensively utilized to design steering controllers capable of maintaining lane stability and achieving precise trajectory following. For example, (Liu and Cui, 2024) formulated the path-tracking problem as a nonlinear optimization task solved through constrained algorithms, while (Rafaila and Livint, 2016) applied H-infinity techniques to develop robust steering controllers that compensate for external disturbances and model uncertainties. Similarly, (Wang et al., 2025) integrated optimal control methods with intelligent vehicle systems to enhance stability and accuracy under varying driving conditions.

Despite its strong theoretical foundation, two major challenges have persisted in LQR controller design since its inception: the solution of the Algebraic Riccati Equation (ARE) and the selection of appropriate weighting matrices (Q) and (R) [S. Das et al., 2013). The quality of these matrices directly influences the control response, sensitivity, and robustness. Moreover, the ARE corresponds to an infinite-horizon control problem, while the DRE formulation provides solutions over finite horizons (B. D. Anderson and J. B. Moore, 2007), (D. E. Kirk, 1998), (D. Choi et al., 2023). For time-invariant systems, the control gain stabilizes to a constant value, simplifying implementation. However, when system matrices vary with operating conditions—such as vehicle speed—the gain must adapt accordingly, often requiring a gain-scheduled LQR framework.

When applied to ground vehicles, traditional LQR design faces additional limitations. The method assumes an accurate linear model, yet vehicle dynamics are inherently nonlinear and time-varying due to tire–road interactions, lateral slip, and load transfer effects. Linearized "bicycle" models are valid only within small operating ranges—typically low speeds and moderate steering angles. Under aggressive maneuvers such as sharp cornering or emergency lane changes, these assumptions break down, causing notable deviations between model predictions and actual behavior. Although gain-scheduling can partially address these issues by computing separate gains for different velocities, it increases design complexity and computational burden.

The finite-horizon approach, based on the DRE, offers an effective alternative. By specifying a terminal time, it enables the adjustment of control aggressiveness and the acceleration of the system's transient response. Expanding the terminal weighting matrix enhances responsiveness by amplifying control inputs near the end of the maneuver (L. G. Lin et al., 2023). Various numerical techniques have been proposed for solving the DRE, including backward-time integration (M. H. Korayem and S. and Nekoo, 2015), Lyapunov-based methods (A. Heydari and S. Balakrishnan 2015), and matrix state-evolution formulations (J. Jung et al., 2013), (M. H. Korayem and S. R. Nekoo, 2014).
In this research, the Finite-Horizon Linear-Quadratic Tracking (FHLQT) method is developed and compared with the classical LQR for a ground-vehicle lane-change application. The study investigates how each controller handles trajectory tracking, robustness, and sensitivity to model variations.

The main contributions of this research can be summarized as follows:
- A comparative study is developed between the classical LQR and the FHLQT controllers for autonomous ground vehicle lane-change maneuvers. This framework highlights the fundamental differences between infinite- and finite-horizon optimal control strategies.
- The paper presents a detailed mathematical formulation of the FHLQT control law for linear time-varying systems. The controller is derived using the Pontryagin Minimum Principle (PMP) and solved through backward integration of the DRE to obtain time-varying optimal gains.

- Integration with Vehicle Dynamics: a nonlinear bicycle model of vehicle dynamics is developed and linearized for controller synthesis. This model accurately captures the lateral and yaw dynamics of ground vehicles during transient maneuvers, serving as a realistic platform for evaluating control performance.
- Extensive simulations are conducted to assess the performance of both LQR and FHLQT controllers under identical operating conditions. The analysis considers several performance indices, including root-mean-square (RMS) tracking error, yaw rate response, and steering smoothness.
- Demonstration of FHLQT Superiority: the results demonstrate that the FHLQT controller achieves substantially improved performance compared with the conventional LQR, with approximately 78% reduction in RMS lateral tracking error, smoother steering action, and enhanced robustness to parameter variations. These findings establish the effectiveness of FHLQT in handling short-duration dynamic maneuvers such as lane changes.
- Practical Implications for Autonomous Driving: the study confirms that finite-horizon optimal control offers a promising framework for advanced vehicle steering and path-tracking tasks, improving transient performance without the need for gain scheduling or nonlinear compensation, thereby enhancing the practicality of real-time autonomous control implementation.

The paper is organized as follows:

- Section 2 introduces the finite-horizon control formulation and the associated Riccati-based solution.
- Section 3 describes the dynamic vehicle model used for controller design and simulation.
- Section 4 presents the simulation results and comparative performance analysis.
- Section 5 summarizes the key findings and provides concluding remarks.

## 2. Controller Design: Linear Quadratic Tracking System: Finite Time Scenario

This section presents the formulation of the FHLQT problem for a linear, time-varying system. The objective is to design a control law that compels the system's output to follow a desired reference trajectory while minimizing the total control effort within a finite time interval (M. H. Korayem and S. R. Nekoo, 2014), (M. Athans and P. Falb, 1966). Consider a linear system described by:

$$\dot{x}(t) = Ax(t) + Bu(t),\ \ x(t_0),\ \ x(t_f) \text{ are free, and } y(t) = Cx(t) \tag{1}$$

Here, $x(t)$ represents the *n*-dimensional state vector, $u(t)$ is the *r*-dimensional control input, and $y(t)$ is the *m*-dimensional system output. Let $z(t)$denotes the desired output trajectory. The tracking error is then defined as:

$$e(t) = z(t) - y(t) \tag{2}$$

Let $z(t)$ represent the m-dimensional desired output, with A(t) and the other matrices specified accordingly. Additionally, let *B*(t) and *C*(t) be matrices of suitable dimensions. The primary control objective is to manage the system in such a way that the output $y(t)$ closely adheres to the desired trajectory $z(t)$ throughout the interval [$t_0$, $t_f$], while simultaneously minimizing the related control effort. To minimize this error over the specified time horizon, the following quadratic cost functional is adopted:

$$J = \frac{1}{2}e^T(t_f)Fe(t_f) + \frac{1}{2}\int_{t_0}^{t_f}(e^T(t_f)Qe(t_f) + u^T(t_f)Ru(t_f))dt \tag{3}$$

with $t_f$ defined and $x(t_f)$left unspecified. In this frmework, it is assumed that $F(t_f)$ and $Q(t_f)$ are $m\ \times m$ matrices, both of which are symmetric and positive semidefinite, whereas $R(t)$ is an $r\ \times\ r$ symmetric matrix that is positive definite. The first term penalizes the final tracking error, while the integral term penalizes both instantaneous tracking error and control effort throughout the maneuver. To obtain the solution, the Pontryagin Minimum Principle is systematically employed, adhering to the following steps (B. D. O. Anderson and J. B. Moore, 1990), (F. L. Lewis, 1986), (F. L. Lewis, 1992): Procedure 1: Formulation of the Hamiltonian -Procedure 2: Determination of the optimal control in an open-loop configuration - Procedure 3: Derivation of the dynamics of the state and costate variables- Procedure 4: Establishment of the Riccati equations along with associated vector equations- Procedure 5: Construction of the optimal control in a closed-loop configuration- Procedure 6: Evaluation of the state corresponding to optimal performance- Procedure 7: Computation of the  minimum achievable cost

To obtain the optimal control input, the Pontryagin Minimum Principle (PMP) is applied. The Hamiltonian is defined as:

$$H(x,u,\lambda) = \frac{1}{2}[z(t) - c(t)x(t)]^T Q(t)[z(t) - c(t)x(t)] + \frac{1}{2}u^T(t)Ru(t) + \lambda^T[Ax(t) + Bu(t)] \quad (4)$$

Using the Hamilton, the control equation $u^*(t)$ from $\frac{\partial H}{\partial u} = 0$, is expressed as

$$u^*(t) = -R^{-1}B^T\lambda^*(t) \quad (5)$$

The objective is to formulate a control law that reduces the cost function *J*. The dynamics of the state are articulated via the Hamiltonian framework.

$$\dot{x}(t) = \frac{\partial H}{\partial \lambda} = Ax(t) + Bu(t) \quad (6)$$

The optimal state equations become

$$\dot{x}^*(t) = Ax^*(t) + Bu^*(t)\lambda^*(t) \quad (7)$$

and the optimal costate equation becomes

$$\dot{\lambda}^*(t) = -\frac{\partial H}{\partial x} = -C^TQC\ x^*(t) - A^T\lambda^*(t) - C^TQC\ z(t) \quad (8)$$

Define $E = B\,R^{-1}(t)B^T$ ,$V = C^TQC,\ W = E = C^TQ$ and by combining the state and by utilizing the costate equations, one can formulate the Hamiltonian canonical system as

$$\begin{bmatrix}\dot{x}^*(t)\\ \dot{\lambda}^*(t)\end{bmatrix} = \begin{bmatrix}A & -E\\ -V & -A^T\end{bmatrix}\begin{bmatrix}x^*(t)\\ \lambda^*(t)\end{bmatrix} + \begin{bmatrix}0\\ W(t)\end{bmatrix}z(t) \quad (9)$$

Considering $W(t)z(t)$ as the driving function, this canonical system consists of $2n$ differential equations and is characterized as linear, time-varying, and nonhomogeneous. The boundary conditions for both the state and costate equations are established by the initial condition applied to the state, $x(t_0)$, in conjunction with the terminal condition imposed on the costate at the final time, $t_f$. Given that the terminal state $x(t_f)$ remains unconstrained, it is regarded as a free variable within the formulation.

$$\lambda(t_f) = C^TF(t_f)C\,x(t_f) - C^TF(t_f)C\,Z(t_f) \quad (10)$$

The boundary condition along with the solution of the system suggests that there exists a linear relationship between the state and the costate as

$$\lambda^*(t) = P(t)x^*(t) - g(t) \quad (11)$$

At this stage, the canonical system is not fully satisfied by the $n \times n$ matrix $P(t)$ and the *n*-dimensional vector $g(t)$. This leads to the requirement that the matrix differential Riccati equation (DRE) be satisfied by the matrix $P(t)$.

At this point, the canonical system is not entirely fulfilled by the $n \times n$ matrix $P(t)$ and the *n*-dimensional vector $g(t)$. Consequently, it is necessary for the matrix DRE to be satisfied by the matrix $P(t)$.

$$\dot{P}(t) = P(t)A - -A^TP(t) + P(t)E(t)P(t) - V(t) \quad (12)$$

In order to satisfy the vector differential equation, the *n-vector* $g(t)$ must fulfill the vector differential equation.

$$\dot{g}(t) = [P(t)E - -A^T]g(t) - W(t)z(t) \quad (13)$$

With the specified boundary condition

$$P(t_f) = C^T(t_f)\, F(t_f)\, C(t_f) \quad (14)$$
$$g(t_f) = -C^T(t_f) F(t_f)\, z(t_f) \quad (15)$$

As illustrated in Fig. 1, the optimal control is presently defined in relation to the state.

$$u^*(t) = -R^{-1}B^T[px^*(t) - g(t)] = [-K(t)x^*(t) + R^{-1}B^T g(t)] \quad (16)$$

Where $K(t) = R^{-1}B^T p(t)$ represent the ***Kalman gain***

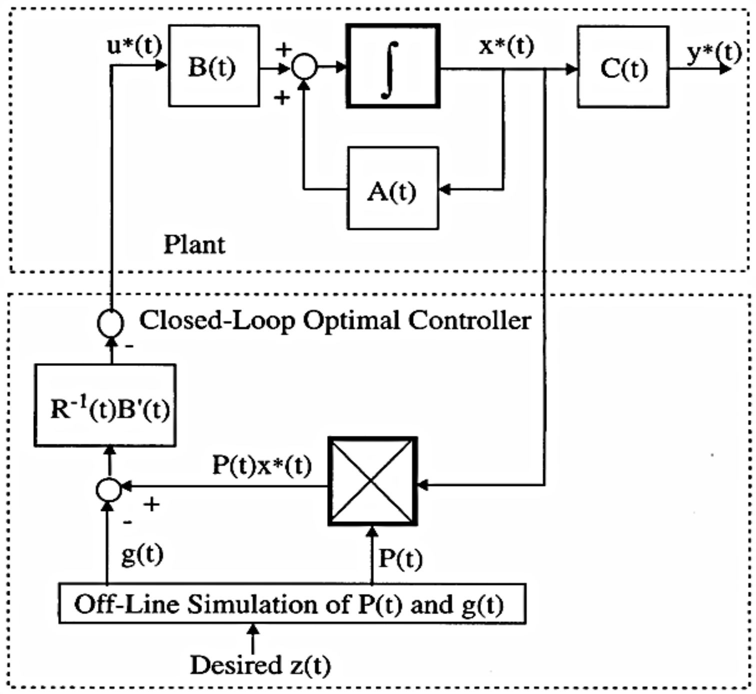


Figure 1. Optimal tracking system with finite-horizon control.

The resulting FHLQT controller continuously updates its feedback gain $K(t)$according to the evolving solution of the Riccati equation. This mechanism enables the controller to handle transient behavior effectively and maintain close adherence to the desired reference trajectory over a finite duration. The overall control structure is summarized schematically in Figure 1, which illustrates the relationship between the plant dynamics, the reference signal, and the time-varying optimal control loop.

## 3. System Description

This section presents the mathematical modeling of a ground vehicle executing a step lane-change maneuver, a typical scenario used to evaluate the performance of steering controllers. During such a maneuver, the vehicle undergoes both translational and rotational motion. To describe these dynamics accurately, a coordinate frame fixed to the vehicle body is adopted, as it allows the moments of inertia to remain constant during motion. The coordinate conventions follow the definitions established by the Society of Automotive Engineers (SAE) (R. N. Jazar, 2008).
As illustrated in Figure 2, vehicle rotations about the principal axes are defined as follows (L. L. P. Soares, 2005):

- Rotation around the x-axis represents roll motion of the vehicle.
- Rotation about the *y*-axis corresponds to pitch.
- Rotation about the *z*-axis represents yaw motion.

The modeling approach follows principles similar to those in the works (J. Y. Wong, 1993) and (A. B. Will and S. H. Zak 1997), but is reformulated here for clarity and analytical simplicity.

For planar motion, the vehicle is assumed to be laterally symmetric with respect to its longitudinal axis. Accordingly, a bicycle model is employed to represent the combined dynamics of the front and rear axles, as shown in Fig. 3. In this configuration, the left and right wheels of each axle are represented by a single equivalent wheel that transmits the combined lateral and longitudinal forces. The main variables and parameters are defined as follows:

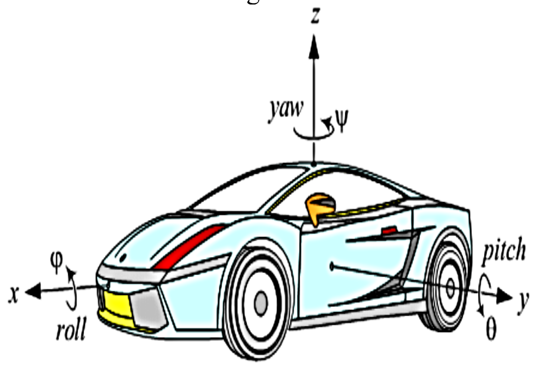


Figure 2. SAE coordinate system.

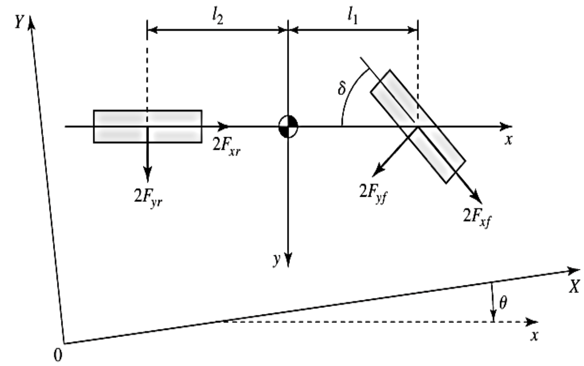


Figure 3. Bicycle model of a vehicle.

For planar motion, the vehicle is supposed to be laterally symmetric relative to its longitudinal axis. Accordingly, a bicycle model is employed to represent the combined dynamics of the front and rear axles, as shown in Fig. 3. In this configuration, the left and right wheels of each axle are represented by a single equivalent wheel that transmits the combined lateral and longitudinal forces. The main variables and parameters are defined as follows:

| | |
|---|---|
| $\delta_f$ : | steering angle of the front wheels |
| $\delta_r$ : | steering angle of the rear wheels |
| $2F_{xf}$ , $2F_{xr}$ : | combined longitudinal forces on the front and rear axles, respectively |
| $2F_{yf}$, $2F_{yr}$ : | combined lateral forces on the front and rear axles, respectively |
| $I_z$: | vehicle's moment of inertia about the vertical ($z$) axis |
| $v_x = \dot{x}$ : | longitudinal velocity along the x-axis |
| $C_{\alpha f}$ , $C_{\alpha r}$: | front and rear tire cornering stiffness coefficients |
| $\psi$: | yaw angle of the vehicle relative to the inertial frame |

A schematic representation of the moving coordinate frame attached to the vehicle is shown in Figure 4, while Figure 5 depicts the vehicle heading angle relative to the global *X–Y* coordinates.

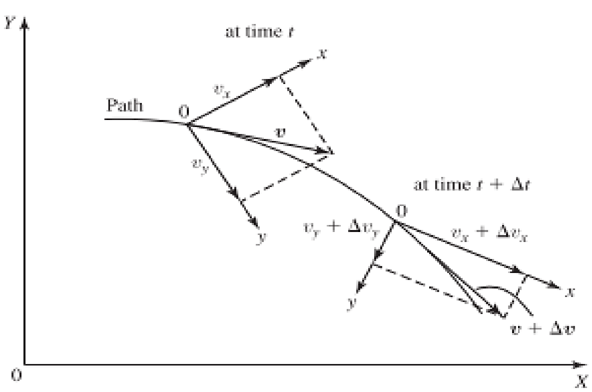


Figure 4. Analyzing vehicle motion using a coordinate frame attached to the vehicle.

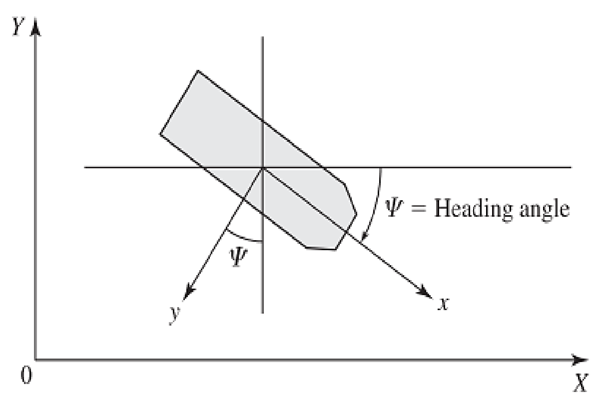


Figure 5. The heading angle of a vehicle in relation to the fixed coordinates *X-Y*.

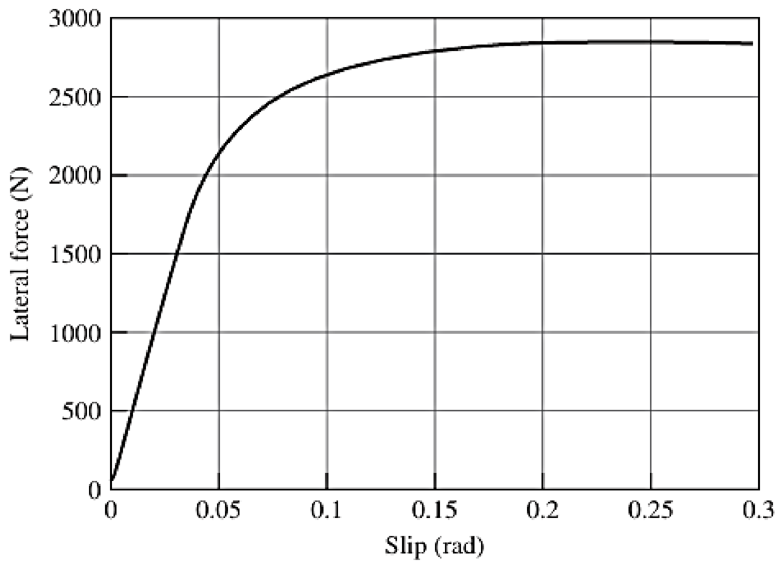


Figure 6. Typical cornering characteristic.

Under the assumption of small slip and steering angles (i.e., sin δ ≈δ, cosδ≈1), the dynamic equilibrium equations along the longitudinal, lateral, and yaw directions can be expressed as:

$$m(\ddot{x} - \dot{y}\omega_z) = 2F_{xf}cos\,\delta_f - 2F_{yf}sin\,\delta_f + 2F_{xr}cos\,\delta_r + 2F_{yr}sin\,\delta_r \approx 2F_{xf} + 2F_{xr} \quad (17)$$

$$\mathrm{m}(\ddot{y} + \dot{x}\omega_z) = 2F_{xf}\ \sin\,\delta_f + 2F_{yf}\ cos\,\delta_f + 2F_{yr}\ \cos\,\delta_r - 2F_{xr}\ \sin\ \delta_r \approx 2F_{yf} + 2F_{yr} \quad (18)$$

Furthermore, by referring to Figure 3 again and presuming small angles, the moment balance about the vertical axis (yaw motion) gives:

$$I_z\dot{w}_z = 2l_1F_{yf}cos\,\delta_f + 2l_1F_{xf}sin\,\delta_f - 2l_2F_{yr}cos\,\delta_r + 2l_2F_{xr}sin\,\delta_r \quad (19)$$

Where $l_1$ and $l_2$ represent the distances between the vehicle's center of gravity (CG) and the front and rear axles, respectively.

The coefficients $C_{\alpha f}$ and $C_{\alpha r}$ are determined through experimental measurements. Assuming small angles as illustrated in Figure 4 and Figure 5, Figure 6 presents a typical relationship between the tire slip angle and the cornering force. This relationship is nonlinear, with the cornering coefficient being

$$C_{\alpha w} = \frac{dF_{yw}}{d_{\alpha w}}\ , \qquad w = \{f, r\} \quad (20)$$

The lateral forces $F_{yf}$ and $F_{yr}$ depend on the respective tire slip angles $\alpha_f$ and $\alpha_r$, as well as the cornering stiffness coefficients $C_{\alpha f}$ and $C_{\alpha r}$. Suppose that $C_{\alpha f} = C_{\alpha r}$ equal constant, and the relationships between the tire slip and lateral forces angles are represented as

$$F_{yf} = C_{af}\,\alpha_f\ and\ \ F_{yr} = C_{ar}\,\alpha_r \quad (21)$$

The lateral and longitudinal motions in the global coordinate frame are related by:

$$\dot{Y} = -\dot{x}\,sin\,\psi - \dot{y}\,cos\,\psi \approx -v_x\psi - \dot{y} \quad (22)$$

$$\dot{X} = \dot{x}\,cos\,\psi - \dot{y}\,sin\,\psi \quad (23)$$

For a step lane-change input, the vehicle's yaw rate corresponds to the yaw rate error; specifically, $w_z = \dot{\psi}$
Additionally, assume that $\theta = \psi$.
To express the model in state-space representation, the following state variables are introduced:

$$x_1 = \dot{y},\ \ x_2 = \theta,\ \ x_3 = w_z,\ \ x_4 = Y \quad (24)$$

The resulting linearized state-space representation becomes:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix} -\frac{2\ C_{\alpha f}+2\ \ C_{\alpha r}}{m\, v_x} & 0 & -v_x-\frac{2l_1C_{\alpha f}-2l_2C_{\alpha r}}{m\, v_x} & 0\\ 0 & 0 & 1 & 0\\ -\frac{2l_1C_{\alpha f}-2l_2C_{\alpha r}}{I_z\, v_x} & 0 & -\frac{2l_1^2C_{\alpha f}-2l_2^2C_{\alpha r}}{I_z\, v_x} & 0\\ -1 & -v_x & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}\frac{2C_{\alpha f}}{m} & \frac{2C_{\alpha r}}{m}\\ 0 & 0\\ \frac{2l_1C_{\alpha f}}{I_z} & -\frac{2l_2C_{\alpha r}}{I_z}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_f\\ \delta_r\end{bmatrix} \quad (25)$$

This formulation provides a representation of the lateral–yaw coupling that dominates the dynamics during lane-change maneuvers. It serves as the foundation for the subsequent design of optimal controllers (LQR and FHLQT), which aim to regulate steering inputs to achieve the desired lateral displacement Y while maintaining vehicle stability.

## 4. Simulation Results

The vehicle and controller parameters are summarized in Table 1. These include the vehicle's mass, yaw inertia, cornering stiffness coefficients, and the geometric distances from the center of gravity to the front and rear axles. A constant longitudinal velocity of $v_x$ was assumed throughout the maneuver to isolate the effects of lateral and yaw dynamics, where $\delta_r = 0$. In other words, our objective is to develop a tracking controller for a vehicle that utilizes front-wheel steering $\delta_f$.

### 4.1 LQR Controller Performance (Fixed Gain)

Utilize the model and data presented in Table 1 for standard data, employing the nonlinear cornering characteristic illustrated in Figure 6, to develop an LQR represented by the equation form $\delta_f = -Kx + Y_{ref}$ where $Y_{ref}$ denotes the command input.

Table 1. Vehicle and controller parameters used in the simulations.

| Parameter | Value | Unit |
|---|---|---|
| Distance from the CG to the front axle $l_1$ | 1.20 | $m$ |
| Distance from the CG to the rear axle $l_2$ | 1.22 | $m$ |
| vehicle mass $m$ | 1,280 | kg |
| Yaw inertia $I_z$ | 2,500 | $kg.m^2$ |
| longitudinal velocity along the $x$-axis $v_x$ | 18.3 | $m/s$ |
| front and rear cornering stiffness and rear tires $C_{\alpha f} = C_{\alpha r}$ | 30,000 | $N/rad$ |

When the elements of the performance index are taken into account, this type of method can yield the most favorable results. To enhance road handling and ensure passenger comfort, the LQR method for vehicle dynamics has been suggested. The mathematical representation of the quadratic cost function is articulated as follows:

$$J = \int_{t_0}^{\infty}(x^T(t_f)Qx(t_f) + u^T(t_f)Ru(t_f))dt \quad (26)$$

In other words, the form will be present in this closed-loop system.

$$\dot{x} = (A - BK) + B\, Y_{ref} \quad (27)$$

In this research, the dynamics of the vehicle are modeled to include the nonlinear cornering characteristics depicted in Figure 6. The results from the simulation of the planar vehicle model, which utilizes LQR state-feedback control with fixed gains $K$= [0.0963 10.6422 1.8047 -1.00], are shown in Figure 7. These results encompass the reference trajectory in response to a step input of steering angle, the vehicle's yaw rate response, and its performance during a lane-change maneuver. The gain $K$ defines the optimal control input ($u$) as a linear function of the current state variables ($x$), represented by the equation $u = -Kx$(t). The control input (steering angle) remains within feasible operational limits, showing that the design provides a realistic steering demand. Nevertheless, the constant-gain structure limits the controller's adaptability, resulting in slightly degraded performance when dynamic variations become more significant near the end of the lane change.

## 4.2 Time Varying LQR with Finite Horizon

The system matrices ($A$ and $B$) along with the cost function weighting matrices ($Q$ and $R$) are subject to change over time. This adaptability is crucial for vehicles, as their dynamic characteristics and the significance of particular states or control efforts may fluctuate based on factors such as speed, road conditions, or the specific phase of a maneuver (for instance, cornering versus straight driving). The objective of designing an optimal regulator is to establish the optimal control law $u(t) = -k(t)x(t)$ which facilitates the transition of the system to move from its initial state to the final state (with zero control input) while minimizing a specified performance index. This performance index is chosen to provide the most favorable balance between performance and control costs. The simulation results demonstrate variations in the lane change maneuver $k(t)$and the state response $x(t)$of the vehicle model system over a finite time period (5 seconds), as illustrated in Figure 8.

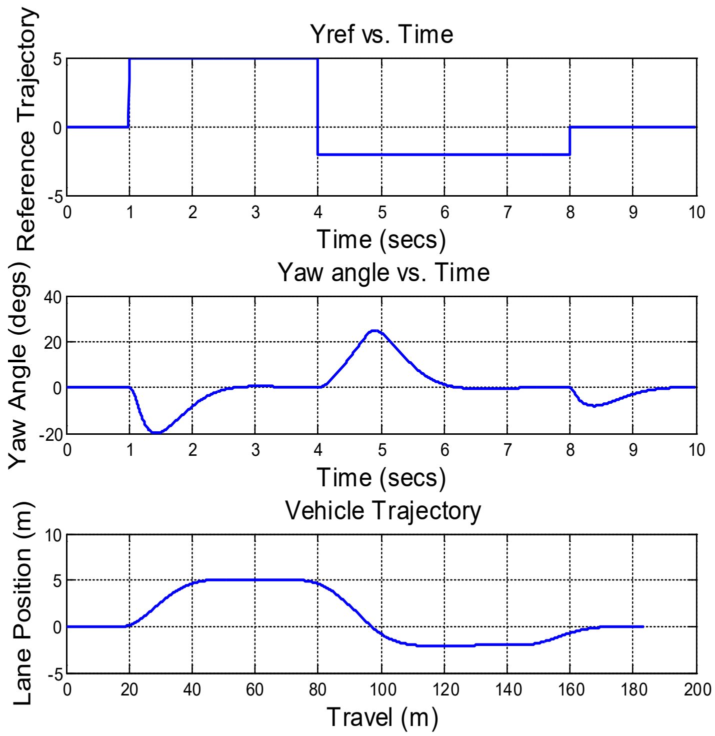


Figure 7. Performance of the vehicle system for $Y_{ref}$ command input.

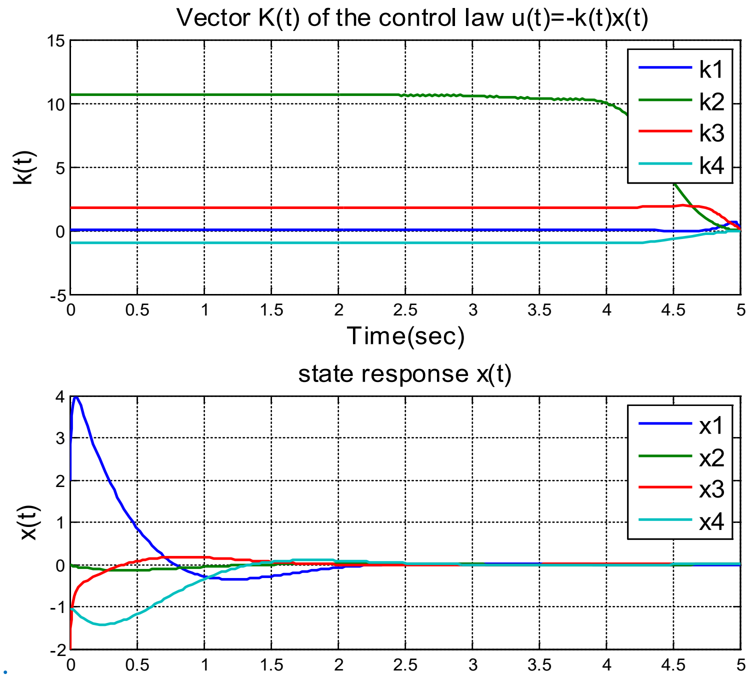


Figure 8. Response of states $x$(t) and controller gain $K$(t).

### 4.3 FHLQT Controller Performance

Unlike the conventional LQR, the FHLQT controller computes time-varying feedback gains by integrating the DRE backward from the terminal time $t_f$ . This allows the controller to update its gain continuously as the maneuver evolves, improving responsiveness to changing system states. The parameters along with their corresponding values are presented in Table 1. The lateral forces, denoted as $F_{yf} = F_{yf}(\alpha_f)$ and $F_{yf} = F_{yr}(\alpha_r)$ are characterized as non-linear functions of their corresponding slip angles, with a typical graph illustrating the relationship between lateral force and tire slip angle depicted in Figure 6. The slip angles for both the front and rear tires are.

$$\alpha_f = \delta - \frac{l_1 x_3 + x_1}{v_x} \quad \text{and} \quad \alpha_r = \frac{l_2 x_3 - x_1}{v_x} \tag{28}$$

A crucial component of automated vehicle control is the step lane-change maneuver, which is essential for cruising operations, The optimal control $u^*(t) = -R^{-1}B^T[px^*(t) - g(t)] = [-K(t)x^*(t) + R^{-1}B^T g(t)]$ exhibits fluctuations over time, as illustrated in Figure 9. This methodology entails planning a desired lane-change trajectory (referred to as the "step") and subsequently employing the FHLQT to produce control inputs (such as steering angle and acceleration/braking) that enable the vehicle to accurately and safely follow this time-varying reference path over a specified duration. Simulation results demonstrate that the FHLQT achieves significantly smaller lateral errors and a smoother transient response compared with the LQR. The yaw-rate and lateral-velocity profiles converge more rapidly to the desired trajectory, and the steering command varies gradually, avoiding abrupt inputs. Figures 7, 8, and 9 illustrate that the FHLQT trajectory closely matches the reference, while the LQR shows a noticeable offset at the end of the maneuver. These results highlight the benefit of finite-horizon optimization in improving tracking precision and dynamic response. The tracking accuracy and steering effort for both controllers were quantified using the Root-Mean-Square (RMS) tracking error, peak yaw rate, and maximum steering input.

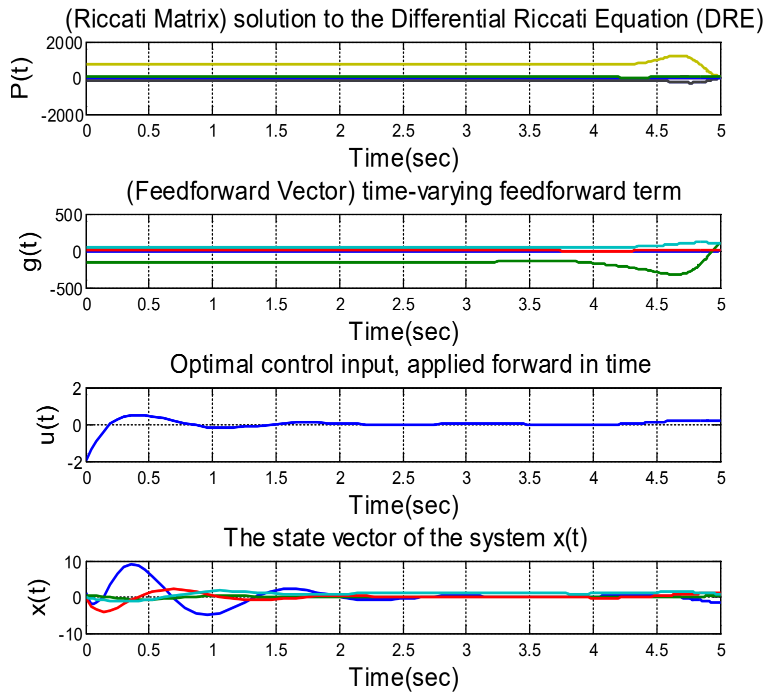


Figure 9. (FHLQT) controller while tracking desired trajectory within 5 sec.

A comparative analysis of FHLQT and the LQR controllers is presented in Table 2. The results show that the FHLQT controller consistently achieves lower RMS tracking error and smoother steering action.

Table 2. A comparative analysis of LQR and FHLQT.

| | **LQR** | **FHLQT** |
|---|---|---|
| RMS lateral error (*m*) | 2.29 | 0.514 |
| Maximum steering input (*deg*) | 6.9 | 1.03 |
| Objective | Optimal regulation (stabilization to zero error) over an infinite horizon. | Optimal tracking of a specific trajectory over a finite horizon. (5 *sec.*) |
| Horizon | Infinite. | Finite (specific duration of the maneuver). (5 *sec.*) |
| Real-time Performance | Stable but potentially less dynamic; optimal for steady-state control. | Highly effective for transient maneuvers like lane changes less than 5 *sec.* |
| Advantage | A constant control gain K may be applied to each trajectory. [0.0963 10.6422 1.8047 -1] | Decreased energy consumption (Fig. 9) |
| Disadvantage | - Increased energy consumption. - Less accurate tracking. (Fig. 7) | Each trajectory necessitates the offline computation of time-variant control gains. Control gains inherently encompass information regarding the desired trajectory. |

## 5. Conclusions

This paper presents a comparative study that leads to the following:

1. Both LQR and FHLQT controllers provide stable steering behavior for lane-change maneuvers.
2. The results show that the FHLQT controller consistently achieves lower RMS tracking error and smoother steering action by dynamically adjusting its feedback gains.
3. The LQR remains computationally efficient but lacks the adaptability required for rapid or nonlinear driving conditions.
4. Finite-horizon optimization enhances system performance without resorting to explicit gain scheduling or nonlinear compensation.

Overall, the results confirm that the FHLQT approach provides a practical and effective framework for advanced vehicle control tasks where transient behavior and path-tracking accuracy are critical.

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