

# **Small Area Estimation of Class-Specific Proportions in Ordinal Categories Using Mixed Effect Cumulative Link Mixed Models**

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## **Abstract**

Most of the studies on small area estimation (SAE) focused on the quantitative, count, and categorical variables. However, in practical policy making problems we also require information on class-specific proportions of an ordinal response variable in small areas. This study proposed a model-based approach for estimating class-specific proportions in small areas using unit level cumulative linked mixed models (CLMMs). The CLMMs based SAE method captures the ordering structure of the categories as well as the nested structure of the data to capture between area variability. The suggested CLMMs based method was compared with the direct method using a simulation as well as study on immunization coverage estimation in different districts of Pakistan using Pakistan Demographic and Health Survey (PDHS) 2017-18 data. The CLMM-based SAE methods outperform the direct method in terms of all performance measures used in this study.

## **Keywords**

Small Area Estimation, Cumulative Link Mixed Models, Ordinal Response Variable, Model-Based Estimation, Demographic and Health Survey.

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## **Biographies**

**Muhammad Hamza** is a Ph.D. candidate in Bio & Medical Big Data under the BK21 Four Program at the Graduate School of Gyeongsang National University, South Korea, where he works under the supervision of Professor Youngsoo Kim and in collaboration with Dr. Shakeel Ahmed. He received the B.S. degree in Statistics from the University of Haripur, Pakistan, in 2018, and the M.S. degree in Statistics from the National University of Science and Technology, Islamabad, Pakistan, in 2022, completing a thesis on a double-goal CART approach for domain-specific health outcome estimation in Pakistan under supervision of Dr. Shakeel Ahmed. From January to August 2023, he served as Biostatistician at Skytech Consultants Inc. Canada, performing and interpreting statistical analysis of healthcare datasets. Earlier, he was a lecturer at Department of Mathematics at Army Public School and College Kohat Cantonment, Pakistan (2019-2020). His research interest includes Biostatistics, Big Data, small-area estimation, model-based and model-assisted survey sampling, machine learning, and the study of income and health inequality with measures such as concentration indices and norm-based indices.

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