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Modeling the Improvement of Goat Meat Quality and Production Through Back-Crossing Breeding Strategy

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Abstract

The global demand for high-quality sheep meat and goat meat has driven innovations in breeding strategies to enhance productivity and profitability. Back-crossing, a selective breeding technique, has emerged as a promising approach to gaining desirable traits while maintaining genetic integrity. This study investigates the application of mathematical models to understand back-crossing strategies in goat breeding, focusing on maximizing meat yield and quality of goat meat. Using a combination of deterministic modeling and macro genetics principles, we developed a framework to predict the inheritance patterns of the overall key traits. Preliminary results demonstrate that a mathematical model can significantly enhance the understanding of outcomes of back-crossing programs. For instance, simulations suggest that strategic selection of back-crossing parental lines is needed to accelerate the introgression of target traits, compared to conventional inter-filial breeding methods. Furthermore, the inclusion of economic factors ensures the feasibility of these improvements for commercial application. This study highlights the potential of mathematical tools to revolutionize goat breeding practices, offering a scientific basis for decision-making in agricultural systems. The findings provide actionable insights for breeders and policymakers, paving the way for sustainable and efficient meat production.

Keywords
Back-Cross Breeding, Mathematical Model, Applied Mathematics, Meat Production and Quality

1. Introduction
Indonesia is one of the countries that has adopted and implemented sustainable development with targets agreed upon by the world and the United Nations, known as the Sustainable Development Goals. There are 17 goals as shown in Table 1.

Table 1. Sustainable Development Goals (source: https://sdgs.un.org/goals)

Goal Number and Issue		Goal Number and Issue
Goal 1: No Poverty Goal 2: Zero hunger Goal 3: Good health and well-being Goal 4: Quality education Goal 5: Gender equality Goal 6: Clean water and sanitation Goal 7: Affordable and clean energy Goal 8: Decent work and economic growth Goal 9: Industry, Innovation, Technology and Infrastructure		Goal 10: Reduced inequality Goal 11: Sustainable cities and communities Goal 12: Responsible consumption and production Goal 13: Climate action Goal 14: Life below water Goal 15: Life on land Goal 16: Peace, justice and strong institutions Goal 17: Partnerships for the goals

These 17 goals are related to each other. Goals 1-4 are closely related to food security, health and education. Here are general explanations of Goal 1 and Goal 2 that are very related to food security issues:

- “SDG 1 aims to eradicate every form of extreme poverty including the lack of food, clean drinking water, and sanitation. Achieving this goal includes finding solutions to new threats caused by climate change and conflict. SDG 1 focuses not just on people living in poverty, but also on the services people rely on and social policies that either promote or prevent poverty." (UNDP-1)
- “SDG 2 is to: "End hunger, achieve food security and improved nutrition, and promote sustainable agriculture." Indicators for this goal are for example the prevalence of diet, prevalence of severe food insecurity, and prevalence of stunting among children under five years of age.” (UNDP-2).

An example of the manifestation of the implementation of SDGs in Indonesia is the Free Lunch Program (PMSG). The program is implemented by providing healthy food and nutrition for all primary and secondary school students so they have healthy bodies, study well, and increase intelligence. To be healthy, students need a nutritious food source that contains various substances the body needs, including carbohydrates, protein, and essential amino acids. Carbohydrates are required as energy that allows us to carry out and maintain daily activities. After carbohydrates, proteins are biomolecules that are very important for life. The main sources of proteins include milk, cheese, meat, eggs, and so on. Protein is important for growth, immunity, and maintaining normal metabolic processes.

Meat is a source of animal protein which is important in supporting nutritional and protein sources for society in general, including in Indonesia. However, meat consumption in Indonesia is still very low compared to the world's average consumption. For example, Indonesia's beef consumption is only 2.66 kilograms (kg) per capita per year, or below the world average of 6.4 kg per capita per year, as stated in the agricultural economics report in the online daily newspaper Republika (Nursyamsi, 2023), especially when compared to the higher average consumption of developed countries in Europe. In this country, health regulations allow meat consumption up to a limit of 250 grams to 500 grams per week, equivalent to approximately 13 kg to 26 kg per capita per year. In theory, with the implementation of PMSG (the ideal), demand for vegetable and animal protein sources, including livestock meat, should increase, because it is very relevant to the goals of PMSG.

Returning to the importance of meat as a source of animal protein and the low average rate of meat consumption in Indonesia, a report explained that according to the President Director of PT Berdikari, "the low consumption of meat cannot be separated from the purchasing power of the community" (Nursyamsi's, 2023). The same report also compared South Korea and Japan, which have high purchasing power, which correlates with high meat consumption. Perhaps this is also one of the causes of the low access to good nutritional intake, both for the general public and pregnant women, which ultimately causes health problems in Indonesia, including many cases of stunting in children. Easy access to nutritional food, such as meat, will contribute in reducing the prevalence of stunting in Indonesia, although the contribution is indirect. Several causes of stunting can be seen in the reference https://www.biofarma.co.id/id/announcement/detail/7-besar-stunting-pada-anak.

Indonesia is a rich country, filled with various biological resources as the source of nutritious food, including livestock such as cows, buffalos, sheep, and goats. These local livestock have the potential to be farmed and used as a source of animal protein. Large-scale farming should be managed optimally to produce nutritious food in large quantities which can be easily accessed by all levels of society at low prices according to economic laws. However, it is also unlucky that many of these local livestock have lower growth quality and lower adult weight compared to imported livestock. Many good quality imported livestock are usually very expensive. For this reason, there has been a lot of thought about improving livestock quality, both in Indonesia and abroad, through various efforts, such as crossbreeding or upbreeding (Shrestha and Fahmy, 2007; Shrestha 2012) and various other strategies (Widyas, 2021).

Daulay (2020) in her book expressed the opinion that "..what is expected from crossbreeding is production that exceeds the average of the two parent breeds, for example in goats what is expected is high growth speed so that young slaughter weight is achieved which is quite high, quality good meat and efficient use of feed as well as good adaptability to the environment. The crossbreeding method is used to obtain individuals with superior production characteristics in a relatively short time." It is explained in her book that among the types of livestock and poultry that are said to be superior are:

- Friesian cattle, Jersey cattle, Hereford cattle, Guernsey cattle and Aberdeen Angus cattle,

- Poll Dorset sheep, Marino sheep, and Leicester sheep,
- Leghorn chickens, Minorca chickens, Light Susex chickens, Barred Plymouth chickens, Rhode Island chickens.

Apart from the livestock above, there are other superior livestock animals, namely superior goats. There are two types of superior goats, meat types (eg Boer) and milk producers (eg Saanen). One idea is to improve the quality of local animals through cross-breeding with superior livestock, for example, cross-breeding between Boer stud goats with local Jawarandu or Kacang goats. In this regard, Ibadurrohman (2017) has conducted research comparing the quality of beef, lamb, and goat meat with the following results:

1. Boer goats are a breed of superior meat goats originating from Africa, which are now being introduced to Indonesia. Its advantages include rapid development, with an average number of 2 children, with rapid body growth that can reach a body weight of 130 kg for males and 75 kg for females.
2. The quality of Boer goat meat (and its cross-breeds) is categorized as good like other types of goat, with an average protein content of 19.43 to 20.18 grams per 100 grams of meat and a relatively low cholesterol content of 63.77 milligrams per 100 grams of meat.
3. Compared with lamb and beef, the protein content of Boer goat meat is almost equivalent, but it has a lower cholesterol content.

Furthermore, Ibadurrohman (2017) concluded that Boer goats are suitable for breeding as a source of animal protein. Even though the price of males is still very costly (around 40 million Rupiah per head of full-blood), the back-cross breeding of Boer goats is feasible by using female local goats (Kacang goats) inseminated by the sperm from superior Boer goats. In Indonesia, the cement of male Boer goats is already available in the market. The cement is available from the artificial insemination center, e.g., BIB Lembang. In this regards, livestock animals, especially goats, not only have an important role as a source of food/animal protein but also play a significant role in the Indonesian livestock sector and the economy of rural communities. Apart from being consumed, these animals are bred as a commodity to earn income. Boer goats are known for their fast growth and high meat productivity, while local goats have excellent environmental adaptability. With the back-cross breeding, it is hoped to produce superior offspring by combining the best genetic characteristics of each type. Hence, back-cross breeding will enhance the quality of the livestock and also the economic values of the animal.

Managing the crossing process is often carried out without careful planning. Inspired by the work of Ibadurrohman (2017) above, we propose a mathematical model for back-cross breeding of Boer goats and local goats (e.g., Kacang goats). Specifically, this research aims to develop a mathematical model to understand and analyze the back-crossing of Boer goats with local less superior goats, in the context of improving the genetic characteristics of the local goat. This model illustrates where the role of mathematical models becomes important because these models can help predict the results of crosses and evaluate the genetic characteristics of the offspring systematically before the back-cross breeding program happens. For instance, simulations suggest that strategic selection of back-crossing parental lines is needed to accelerate the introgression of target traits, compared to conventional inter-filial breeding methods.

1.1 Objectives

The objective of the study is to construct a mathematical model that able

1. to compute the Boer degree of a filial in a back-cross breeding;
2. to predict the properties of the Boer degree of filials in a consecutive back-cross breeding;
3. to determine the number of filial generation for a predetermined required Boer degree and the level of Boer degree improvement of the filial.

2. Literature Review

Worldwide meat production has been high in 2024, coupled with strong demand from key players in the global market. 2024 saw a high level of meat, especially beef, production worldwide, together with robust demand from major global market participants (Smith, 2024). For 2025, Smith (2024) observed the following key factors for the meat demand: cattle supplies are becoming more scarce and expected to keep declining until 2025; meat consumption rises in many big countries while cattle numbers drop, and the result, there is a strong demand for imports from the region. These facts could support the prediction that the future worldwide demand for meat will be high. To satisfy the demand, there should be a way to enhance meat productivity quickly. Among the ways is to provide varieties of meat sources, such as cows, sheep, and goats, with good quality meat and productive breeding, such as Boer goats.

Boer or Boerbok is a kind of meat goat native to South Africa. Beginning around 1920, the Eastern Cape began carefully breeding it for its meat qualities and capacity to survive by grazing on the local thorn veldt. It has been utilized to enhance the meat quality of other breeds and sold to other nations. Back-crossing, a selective breeding technique, has emerged as a promising approach to gaining desirable traits while maintaining genetic integrity. Crossbreeding native goats to improve their genetic makeup is a significant method of rapidly increasing production (Schiermieste, 2014; Williams et al., 2014). As a result, several governmental and non-governmental organizations have helped bring meat-type Boer goats to their countries (Tesema et al., 2020). Boer goats are well known for their excellent meat and quick growth. Native goats, on the other hand, can withstand heat, thirst, and illness. Consequently, crossbreeding is used to combine productivity with flexibility. For instance, some goat breeds of Ethiopia have crossed with the Boer goat [Tesema et al., 2020, 4]. Recently, Elieser et al. (2012) have compared the productivity of the Boer and Kacang goat dams and found that the reproductive performance of Boer and Kacang goats was similar. However, the productivity of the Boer goat was better than the Kacang goats.

Since in general the Boer goat has better productivity, there are some attempts to improve the quality of native goats by cross-breeding with the Boer goats (Habtegiorgis et al., 2024; Widyas et al., 2021; Nugroho et al., 2019). This procedure results in a higher-quality goat, commonly coined as a purebred goat. The goal of this approach is to acquire a superior species. In this method, a male (buck) of an exotic goat with a higher goat quality (full blood) is mated with a female (doe) of a goat of lower quality, as measured by the goat's meat and milk. This kind of breeding, i.e., to breed a native Kacang doe / *Capra hircus* with an exotic male goat (e.g. Boer goat), is now becoming a common practice in Indonesia. Filial 1 (F1) is the name of the first generation's producing child. If the F1 is a female, she will mate with another male exotic goat of the same kind to create an F2 female, who will then mate with another male exotic goat of the same variety to obtain an F3. This procedure continues and as n gets bigger, the quality of Fn gets closer to the quality of Boer parent goat. There are also different approaches to understanding cross-breeding, such as Raihan et al., 2016; Supriatna et al. (2017), and Carnia et al. (2018). Raihan et al. (2016) discuss a new mathematical model of goat breeding strategy, i.e., backcross breeding. In that reference, the authors assumed that, for n = 1, 2, 3, 4, and 5, the resulting quality of the filial follows the sequence 50% (½), 75% (¾), 88% (7/8), 94% (15/16), and 97% (31/32), respectively, compared to the Boer parent full blood goat. However, they did not explain in more detail how this sequence behaves for higher filial nor did they give a mathematical proof for this sequence. In this paper we will discuss the mathematical approach that emphasizes proving the properties of the Boer quality of backcross filial.

3. Methods

The object of this study is the genetic quality of cross-bred livestock, specifically the back-cross bred between Boer goats and local goats. In general, this quality may be seen as physical characteristics and appearance or as genetic properties, such as growth, environmental adaptation, and internal meat productivity. Furthermore, the quality will be defined as the degree of presence of Boer genetic properties in the resulting back-cross bred of the livestock. This quality will be determined through a mathematical modeling approach. Hence, the direct object of study is a mathematical expression. In the following subsections we will explain some mathematical concepts that will be used in the subsequent discussion.

3.1 Mathematical Modeling

What is mathematical modeling? There are several definitions of mathematical modeling in the literature. As an example, it is defined as the process of transforming a real-life problem into a mathematical problem by using a mathematical model. Meyer (2004) defines a mathematical model as "a model whose parts are mathematical concepts, such as constants, variables, functions, equations, inequalities, etc ". Meanwhile, he defines a model as "an object or concept that is used to represent something else". When mathematics is applied to real-life problems, a translation process is needed. This process is often called mathematical modeling. One of the stages in mathematical modeling is simplification. This stage is carried out to produce a model that is easy to solve, but still conveys the essence of the problem being studied (Gunawan in Sibarani (2021)). Furthermore, it is emphasized that the mathematical modeling cycle has stages "including making observations, identifying the main factors involved in the problem being studied, making appropriate assumptions, and carrying out simplifications." Then a temporary relationship between the factors involved should be constructed to build the model. Finally, mathematical tools are used to solve the model, followed by an interpretation of the resulting solution in the original setting of the real problem.

According to Walter J. Meyer's book "Concepts of Mathematical Modeling" (Meyer, 2004), the general steps or stages of mathematical modeling are depicted in Figure 1. These include:

1. Formulation
   a. Defining the question: Ensure the question is specific and well-defined, particularly if it is initially too broad or ambiguous.
   b. Identifying key factors: Determine which quantities or relationships are essential to address the question effectively.
   c. Creating a mathematical framework: Represent each quantity as a mathematical entity (e.g., variable or function) and express relationships through equations or inequalities.
2. Mathematical Manipulation (e.g., solving an equation, ploting the graph, etc.)
3. Evaluation
   Test the model to determine if it produces accurate and reliable results. If the outcomes are unsatisfactory or if the model exhibits limitations, investigate the sources of these issues. This could involve identifying errors in the mathematical manipulation or revisiting and improving the formulation of the model.

Figure 1. shows the flow diagram of mathematical modeling stages (modified from Meyer 2004). In the original Meyer diagram, the concentric circle is the stop button. In this figure, we modify so we can decide to stop or go on to an interpretation or modification stage. Hence, we can consider the mathematical modeling as a cyclical process.

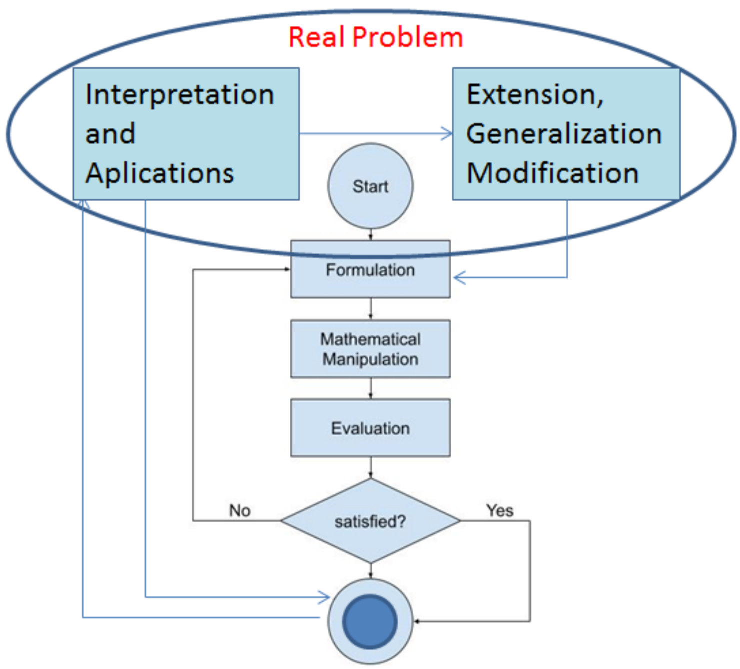

Figure 1. Flow diagram of mathematical modeling stages. It is modified from Meyer (2004).

3.1 Mathematical Induction

The following definition of mathemaatical induction is takeb from (Barnes and Gordon, 1987). Mathematical Induction is a strong and sophisticated method for demonstrating some kinds of mathematical claims, such as general propositions that indicate that something is true for all positive integers or for all positive integers from a given point in time. Examining a few straight forward examples is a logical place to start when demonstrating a lot of mathematical results. This clarifies the assertion and might even provide some guidance on where to go for evidence. One way to approach mathematical induction is to view the claim we are attempting to establish as a series of assertions, one for each n, rather than as a single proposition. Proving the first assertion in a sequence and then demonstrating that, if any given statement is true, the one after it is likewise true is the technique employed in mathematical induction. This allows us to determine that each and every statement is accurate. The general process of mathematical induction can be describe as in Barnes and Gordon (1987) :

The Base Case
Verify that the proposition holds when $n = 1$. (Alternatively, if the proposition claims to be valid for $n \geq a$, demonstrate its truth for n =a.)

The Inductive Step
Show that if the proposition is valid for $n = k$, it must also hold for for $n = k + 1$. In this step, breaking it into smaller stages might be helpful:

- **Stage 1**: Clearly state what the proposition claims for $n = k$. This assumption is known as the *inductive hypothesis*.
- **Stage 2**: Write out what the proposition claims for for $n = k + 1$. This is the statement you need to prove, so keep it in focus.
- **Stage 3**: Using the assumption from Stage 1, prove the statement from Stage 2. There's no universal formula for this step—it depends on the problem at hand.

Similar definitions and examples of proof of mathematical induction can also be seen at Studio Belajar (link available at https://www.studiobelajar.com/induction-mathematics/).

4. Data Collection
Since it is an axiomatic study, we did not collect data for the computation or simulation. Instead, we use hypothetical data or parameters to illustrate the analytical results used for calculation in Subsection 5.1 and generate tables and graphs in Sections 6 and 7.

5. Results and Discussion
Results are derived using the following assumptions:

1. Crossbreeding is carried out between a Fullblood male Boer (notated with FB) with a local female (notated with F_0) which produces a Filial (notated with F_1) with mixed characteristics of half of FB and half of F_0.
2. Back-cross breeding is carried out between another Fullblood male Boer (still denoted FB) with an F_1 female Filial with mixed characteristics of half of FB and half of F_1. The same process is carried out in the same way to produce F_1, F_2, F_3, ... and so on up to F_n (see back-cross diagram in Figure 2).

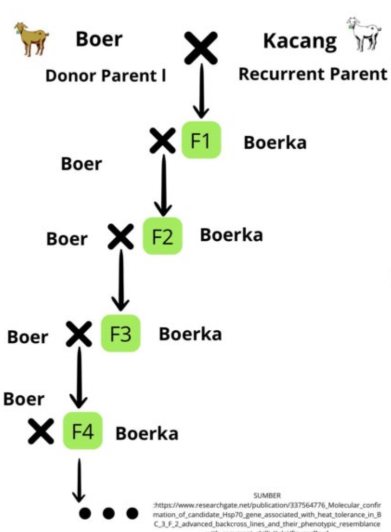

Figure 2. Backcross diagram of a male Boer goat with a female local goat (Kacang) to produce a female Boerka (F_1) and their subsequent filials (modified from Usman et al., 2019).

We derive the aforementioned assumptions from the macro genetic principle, in which in a sexually duplicating life form, one-half of the chromosome complement is paternally acquired. Meanwhile, the remaining half of the complement is maternally acquired. In this view, an individual generally acquires half of their hereditary properties from their male parent and the other half from their female parent. Each parent contributes one set of chromosomes evenly, amid fertilization coming about, in a 50/50 part of hereditary material (Rodakis, 2013). Figure 2 shows the diagram of this principle in the backcross breeding of a male Boer goat with a female local goat (Kacang) to produce a female Boerka (F1). The resulting female Boerka (F1) is then backcrossed with another male Boer goat to produce a female Boerka (F2). Crossing continues in the same pattern to produce Boerka F3, Boerka F4, etc.

In this study, livestock quality is defined as the degree of presence of Boer genetics in parent livestock and cross-bred livestock. If the Boer genetic degree of the Fn animal is denoted by $\delta(\mathrm{F}_n)$, then we obtain
1. $\delta(\mathrm{FB}) = 1$ (the Boer degree of the original Fullblood Boer goat is 100%=1)
2. $\delta(\mathrm{F}_0) = 0$ (the Boer degree of Kacang goat F_0 is 0%=0)

The Boer degree for the following cross (filial) results can generally be obtained as follows.

Filial 1 or F_1:
The Boer Degree of $\mathrm{F}_1 = \delta(\mathrm{F}_1) = \frac{1}{2}(FB) + \frac{1}{2}(F0) = \frac{1}{2}(1) + \frac{1}{2}(0) = \frac{1}{2}$

Filial 2 or F_2:
The Boer Degree of $\mathrm{F}_2 = \delta(\mathrm{F}_2) = \frac{1}{2}(FB) + \frac{1}{2}(F1) = \frac{1}{2}(1) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{3}{4}$

Filial 3 or F_3:
The Boer Degree of $\mathrm{F}_3 = \delta(\mathrm{F}_3) = \frac{1}{2}(FB) + \frac{1}{2}(F2) = \frac{1}{2}(1) + \frac{1}{2}\left(\frac{3}{4}\right) = \frac{7}{8}$

Filial 4 or F_4:
The Boer Degree of $\mathrm{F}_4 = \delta(\mathrm{F}_4) = \frac{1}{2}(FB) + \frac{1}{2}(F3) = \frac{1}{2}(1) + \frac{1}{2}\left(\frac{7}{8}\right) = \frac{15}{16}$

The process of backcrossing can be continued onward. By looking at the correspondence of the *n*-th Filial to the degree of Boer genetics, viz

$$\delta(\mathrm{F}_0) = \frac{0}{1}; \qquad \delta(\mathrm{F}_1) = \frac{1}{2}; \qquad \delta(\mathrm{F}_2) = \frac{3}{4}; \qquad \delta(\mathrm{F}_3) = \frac{7}{8}; \qquad \delta(\mathrm{F}_4) = \frac{15}{16}$$

it can be conjectured that $\delta(\mathrm{F}_5) = \frac{31}{32}$ and furthermore $\delta(\mathrm{F}_n) = \frac{2^n-1}{2^n}$. To prove this conjecture will be done using the proof method of mathematical induction.

Remark: Up to this point we have the following Conjecture 1 which is proved using mathematical induction.

Conjecture 1: *The Boer degree of Boerka goat F_n is given by $\delta(F_n) = \frac{2^n-1}{2^n}$.*

Proof:

Consider the statement $P(n)$: $\delta(\mathrm{F}_n) = \frac{2^n-1}{2^n}$. We have the following results.
Firstly, note that the statement $P(n)$ is true for at least one integer, for example in the calculation above we get the statement $P(n=1)$: $\delta(\mathrm{F}_{n=1}) = \frac{2^{n=1}-1}{2^{n=1}}$, i.e. $\delta(\mathrm{F}_1) = \frac{1}{2}(FB) + \frac{1}{2}(F_0) = \frac{1}{2}(1) + \frac{1}{2}(0) = \frac{1}{2} = \frac{2^1-1}{2^1}$. So, the conjecture is true for $n = 1$.
Secondly, let us assume that the statement is true for $n = k$, namely, $\delta(\mathrm{F}_{n=k}) = \frac{2^{n=k}-1}{2^{n=k}}$.

Next, it will be proven that if the statement $(n = k)$ is true, namely, $\delta(F_{n=k}) = \frac{2^{n=k}-1}{2^{n=k}}$, then the statement $\delta(F_{n=k+1}) = \frac{2^{n=k+1}-1}{2^{n=k+1}}$ is also true. To make it easier, the notation $\delta(F_{n=k}) = \frac{2^{n=k}-1}{2^{n=k}}$ will be shortened to $\delta(F_k) = \frac{2^k-1}{2^k}$, as well as the notation $\delta(F_{n=k+1}) = \frac{2^{n=k+1}-1}{2^{n=k+1}}$ will be shortened to $\delta(F_{k+1}) = \frac{2^{k+1}-1}{2^{k+1}}$. To prove this, let us see the following induction.

$$F_{k+1} = \delta(F_{k+1}) = \frac{1}{2}\delta(FB) + \frac{1}{2}\delta(F_k) = \frac{1}{2}(1) + \frac{1}{2}\left(\frac{2^k-1}{2^k}\right)$$

$$= \frac{1}{2} + \frac{1}{2}\left(\frac{2^k-1}{2^k}\right) = \frac{1}{2}\frac{2^k}{2^k} + \frac{1}{2}\left(\frac{2^k-1}{2^k}\right) = \frac{2^k+(2^k-1)}{2^{k+1}} = \frac{(2^k+2^k)-1}{2^{k+1}} = \frac{2(2^k)-1}{2^{k+1}} = \frac{2^{k+1}-1}{2^{k+1}}.$$

It has been shown above that $\delta(F_{k+1}) = \frac{2^{k+1}-1}{2^{k+1}}$, which means the statement $P(n)$: $\delta(F_n) = \frac{2^n-1}{2^n}$ is true for all n integers with $n \geq 0$. Thus, it can be concluded that the Boer degree of the n-th generation cross is $\delta(F_n) = \frac{2^n-1}{2^n}$, which applies to all integers $n \geq 0$. Consequently we have the following theorem. We also notice that, using the L'Hôpital's Rule, $\lim_{n\to\infty} \frac{2^n-1}{2^n} = 1$.

Theorem 1: *The Boer degree of Boerka goat* F_n *is given by* $\delta(F_n) = \frac{2^n-1}{2^n}$ *with* $\lim_{n\to\infty} \delta(F_n) = 1$.

The complete degree of the Boer genetic is presented up to 20 generations in Table 2 of Subsection 5.1, and its graph is in Figure 3 of Subsection 5.2.

5.1 Numerical Results

In this section, we present a numerical example to illustrate the results in the previous section. Here, we compute the Boer degree of the Boerka, up to 20 generations of back-cross breeding, using the formula in Theorem 1, as shown in Table 2.

Table 2. Boer degree (in 100 %) of the Boerka up to 20 generations of back-cross breeding.

Crossbred Generation index (n)	Filial	Boer Degree of Parent (in 100 %)		Formula 1 of Boer Degree (2^n-1) / (2^n)	Formula 2 of Boer Degree 1 - 1/(2^n)
		Male Boer	Female Boerka		
1	F1	1	0	0.5	0.5
2	F2	1	0.5	0.75	0.75
3	F3	1	0.75	0.875	0.875
4	F4	1	0.875	0.9375	0.9375
5	F5	1	0.9375	0.96875	0.96875
6	F6	1	0.96875	0.984375	0.984375
7	F7	1	0.984375	0.9921875	0.9921875
8	F8	1	0.9921875	0.99609375	0.99609375
9	F9	1	0.99609375	0.998046875	0.998046875
10	F10	1	0.998046875	0.999023438	0.999023438
11	F11	1	0.999023438	0.999511719	0.999511719
12	F12	1	0.999511719	0.999755859	0.999755859
13	F13	1	0.999755859	0.999877930	0.999877930
14	F14	1	0.999877930	0.999938965	0.999938965
15	F15	1	0.999938965	0.999969482	0.999969482
16	F16	1	0.999969482	0.999984741	0.999984741
17	F17	1	0.999984741	0.999992371	0.999992371
18	F18	1	0.999992371	0.999996185	0.999996185

19	F19	1	0.999996185	0.999998093	0.999998093
20	F20	1	0.999998093	0.999999046	0.999999046

5.2 Graphical Results

In this section, we present a graphical presentation of the Boer degree of the Boerka, up to 20 generations of back-cross breeding, using the formula in Theorem 1, as shown in Figure 3.

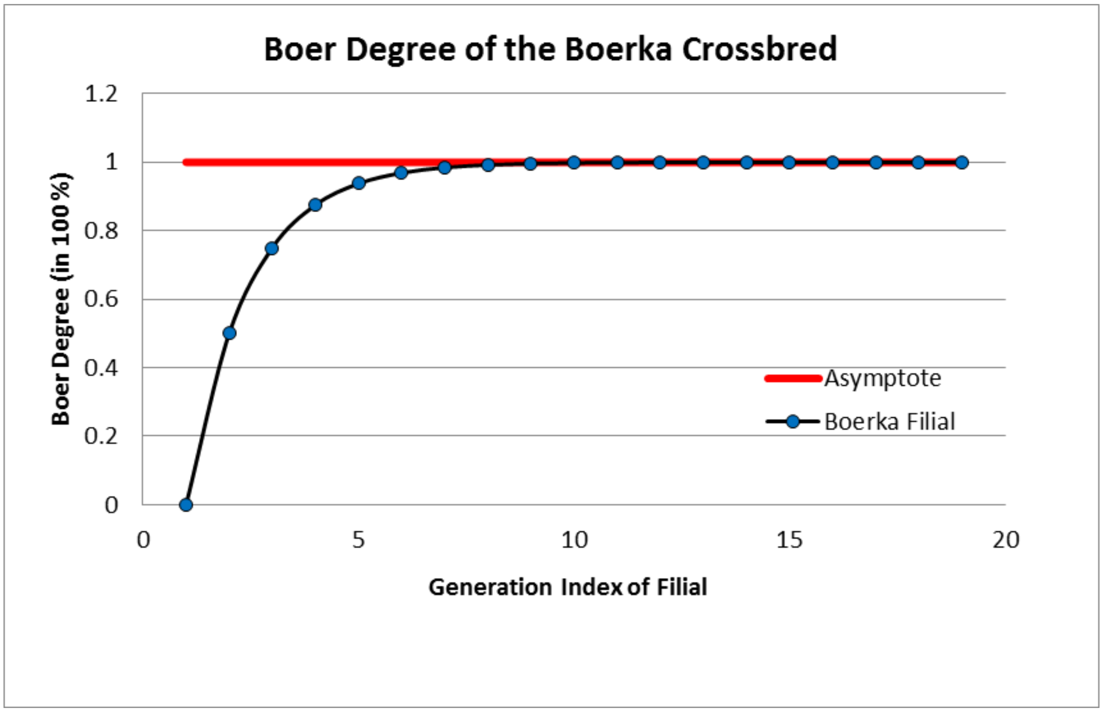

Figure 3. Graph of Boer degree for Boerka filial as a function of the crossbred generation index *n*. The graph is asymptotic to the red line of 100 % for a sufficient large of *n*. The unit of Boer degree is in 100 %.

Figure 3 shows that the graph of Boer degree is an increasing function of the generation index *n* with $\lim_{n \to \infty} \delta(F_n) = 1$, but never reached it. This condition means that the larger the filial, the closer the characteristics of the filial to the properties of the full-blood Boer parent, but their genetics are never the same. Hence, pure breed back-crossed Boerka is never the same as a full-blood Boer, despite their physical characteristics being the same.

5.3 Proposed Improvements

In the model above, we have already had a method to determine the Boer degree of the resulting back-cross breeding for any filial by using Theorem 1. Alternatively, we can find the Boer degree by inspection using the look-up table (Table 2). Theorem 1, Table 2, and Figure 3 suggest that strategic selection of back-crossing parental lines is needed to accelerate the introgression of target traits, i.e., to increase the Boer level of the local (Kacang) goat filial through back-crossing. Next, we will improve the presentation of Table 2 so it could give information on how to determine the generation index of the filial for a predetermined target of Boer degree of the filial and its degree improvement level. Table 3 below gives the level of improvement. Figure 4 shows the declining improvement in Boer Degree as the filial generation index increases. By referring to Table 3, if we set the resulting Boer degree to be a minimum of 95% of the Boer male parent, we have to proceed up to the 5th generation of the back-crossing. Likewise, if we set the resulting Boer degree to be a minimum of 3% improvement of the female parent, then we have to proceed up to the 5th generation of the backcross-breeding (see also Table 3).

Table 3. Improvement level of Boer degree up to 20 generations of back-cross breeding

Crossbred Generation index (*n*)	Filial	Boer Degree of Parent (in 100 %)		Boer Degree Percentage of the Filial Compared to the Boer Full-Blood	Percentage Improvement of the Filial's Boer Degree
		Male Boer	Female Boerka		
1	F1	1	0	50.0000000%	50.00000000%
2	F2	1	0.5	62.5000000%	25.00000000%
3	F3	1	0.75	81.2500000%	12.50000000%
4	F4	1	0.875	90.6250000%	6.25000000%
5	**F5**	**1**	**0.9375**	**95.3125000%** 🡐	**3.12500000%** 🡐
6	F6	1	0.96875	97.6562500%	1.56250000%
7	F7	1	0.984375	98.8281250%	0.78125000%
8	F8	1	0.9921875	99.4140625%	0.39062500%
9	F9	1	0.99609375	99.7070313%	0.19531250%
10	F10	1	0.998046875	99.8535156%	0.09765625%
11	F11	1	0.999023438	99.9267578%	0.04882813%
12	F12	1	0.999511719	99.9633789%	0.02441406%
13	F13	1	0.999755859	99.9816895%	0.01220703%
14	F14	1	0.999877930	99.9908447%	0.00610352%
15	F15	1	0.999938965	99.9954224%	0.00305176%
16	F16	1	0.999969482	99.9977112%	0.00152588%
17	F17	1	0.999984741	99.9988556%	0.00076294%
18	F18	1	0.999992371	99.9994278%	0.00038147%
19	F19	1	0.999996185	99.9997139%	0.00019073%
20	F20	1	0.999998093	99.9998569%	0.00009537%

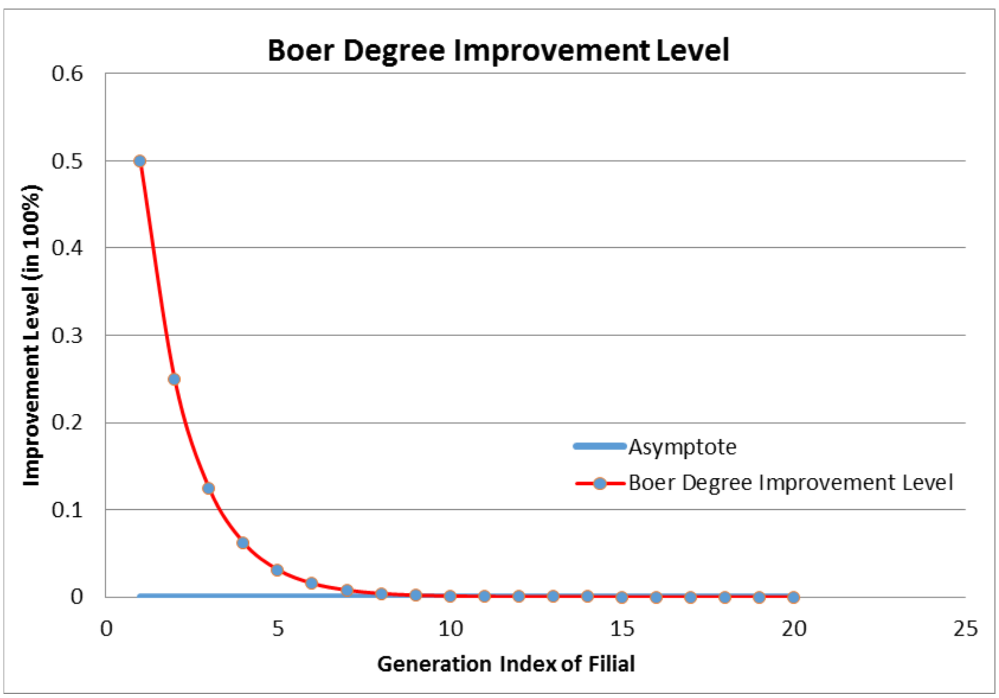

Figure 4. The declining improvement in Boer degree (unit in 100 %) as the filial generation index *n* increases.

6. Conclusion

The present study investigated the application of a mathematical model to understand back-crossing strategies in goat breeding, focusing on maximizing meat yield and quality of goat meat, through the implementation of back-crossing. Using a combination of deterministic modeling and macro genetics principles, we developed a framework to predict the inheritance patterns of the overall key traits. We obtained a mathematical model capable of computing the Boer degree of a filial in a back-crossing. The model can predict the properties of the Boer degree of any filial in a consecutive back-crossing. For instance, the analytical formula and simulations suggest that strategic selection of back-crossing parental lines is needed to accelerate the introgression of target traits, compared to conventional inter-filial breeding methods. In addition, it can also determine the filial generation for a required Boer degree and a certain level of Boer degree improvement of the filial. The back-crossing can be terminated to a certain generation of filial with sufficiently high predetermined Boer degree, e.g., up to F5, since further filial will not give a significant improvement. Moreover, the inclusion of economic factors ensures the feasibility of these improvements for commercial application.

The results above are obtained by combining analytical and numerical methods. Overall, the paper demonstrated that a mathematical model can significantly enhance the understanding of back-crossing programs. Finally, this study highlights the potential of mathematical tools to revolutionize goat breeding practices, offering a scientific basis for decision-making in agricultural systems. The findings provide actionable insights for breeders and policymakers, paving the way for sustainable and efficient meat production.

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