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System Resilience Modeling under Recurrent Hazards

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Abstract

Engineering systems are frequently exposed to a variety of natural and human-made hazards, which can significantly disrupt their operations. Assessing system performance under these recurrent hazards is challenging, especially when these events occur stochastically and influence each other dynamically. In this paper, we introduce an analytical method to model such hazards and use it to evaluate the system's multimodal performance, specifically its availability, and resilience. We derive the joint distribution of hazard occurrences and severities over time, and connect this distribution to system degradation and recovery processes to ultimately determine the system's multimodal availability. A simulation study is conducted to verify the feasibility of the proposed model.

Keywords
Engineered systems, Recurrent hazards, Resilience.

1. Introduction
Throughout the first two decades of the twenty-first century, critical large-scale systems have expanded across multiple sectors. Any disruption or damage to these systems can have serious implications for public health and economic stability. While inherent system failures are typically addressed through reliability analysis and preventive maintenance, external hazards—both natural and manmade—pose significant risks to system functionality. These external hazards require resilience to be used as an evaluation metric, in contrast to inherent failures. Additionally, in many parts of the world, risks often involve multiple concurrent hazards, highlighting the importance of research into multi-hazard resilience. In this paper, we focus on modeling the multi-hazard resilience of an engineered system that faces recurrent and interdependent hazards. The three main research objectives are: identifying the hazards that threaten normal system operations, modeling how the system performs under these hazards, and quantifying the system's resilience based on its performance.

2. Literature Review
To date, numerous studies have focused on the resilience modeling of engineered systems exposed to multiple hazards. These studies cover various aspects, including hazard characterization, system performance under multiple hazards, and resilience modeling in such contexts. A brief review of this body of work is provided below.

There have been substantial scenario-based studies that describe the multi-hazard. Typically, they thoroughly analyze specific hazards in specific regions and do not highlight the mathematical characterization of the random occurrence frequency and severity of the hazards. it has been well recognized that hazards occur randomly in terms of both frequency and severity. To prepare effective hazard mitigation plans, it is essential to develop mathematical methodologies to probabilistically characterize the hazards' occurrence frequency and severity. In current studies, the Poisson (homogeneous or nonhomogeneous) process and Binomial distribution are two main models for hazards' occurrence ((Zobel and Khansa, 2014), (Ouyang et al., 2012), (Hoque et al., 2019) and (Dessavre et al., 2016)) and severity ((Yodo et al., 2017)) characterization. It is also well recognized that hazards exhibit randomness in both frequency and severity, making probabilistic methods crucial for robust mitigation planning. Common probability distributions used to model hazard occurrences include Poisson (both homogeneous and non-homogeneous) and Binomial processes. The severity of these hazards can sometimes be modeled using distributions like the Binomial or other suitable approaches. Researchers typically classify methods to account for hazard interactions into three categories: qualitative, semi-quantitative, and fully quantitative. Qualitative models ((Di Mauro et al., 2006) and (Marzocchi et al., 2012)) offer frameworks and guidelines without heavy numerical detail, while semi-quantitative methods use matrices and event trees ((Kong and Simonovic, 2019) and (Marzocchi et al., 2009)). Quantitative models, whether nonparametric (e.g., random forest, probability maps, Copula functions) or parametric (e.g., joint probability approaches, multiple regression, Bayesian networks), rigorously capture dependency structures among hazards. Building on multi-hazard characterizations, researchers often assess system performance or resilience under particular hazards, with a frequent focus on localized settings. For example, (Preston et al., 2016) examines power grid resilience in the United States by scrutinizing the entire hazard spectrum. Arizona's flood-wildfire-drought mitigation plan likewise addresses multiple perils, though each hazard is mostly considered independently ((Srivastava and Laurian, 2006)). Several probabilistic frameworks have been developed to address the performance modeling and optimization of an individual system (Li et al., 2024, Wei et al., 2024, Gao et al., 2021, Cheng et al., 2021a, Zhao et al., 2025, Wei et al., 2025) or a population of systems (Cheng et al., 2021b) that are subject to multiple sources of stresses or external shocks. Based on these studies, criticality analysis is conducted (Zou et al., 2023b, Wang et al., 2022, Zou et al., 2023a).

Despite some progress in describing dependent multi-hazards (often drawing on historical or simulated data), existing frameworks remain fairly narrow in scope. They usually emphasize specific hazards in particular regions rather than covering the full range of possible hazardous events. Additionally, any interdependence among hazards—when accounted for—tends to be treated as a static parameter, such as through a hazard matrix. This assumption does not accurately represent realistic scenarios where hazards unfold and interact dynamically over time. Furthermore, hazards often occur multiple times, each with random timing and severity, making it complex to capture their interrelations fully. These considerations underscore the need for probabilistic models that depict random hazards—both in occurrence and severity—while incorporating evolving interdependencies over extended time horizons. Although many studies examine performance under a variety of hazards, a common strategy is to treat each hazard independently, then combine the individual performance losses (e.g., via summation or averaging). Such methods do not address scenarios in which hazards arise in a random, interacting fashion that influences how they jointly compromise system functionality. While some dynamic studies exist, they are often simulation-based and do not explicitly handle hazards' repeated occurrences in an analytical manner. Consequently, there is a pressing need for a suitable system performance indicator along with an analytical framework that can capture how dependent, recurring hazards jointly shape overall system performance.

3. Problem Statement

In practical scenarios, critical infrastructures typically consist of geographically dispersed clusters of components or units, each vulnerable to multiple hazards. These hazards—referred to here as "heterogeneous hazards"—may differ in frequency, severity, and their mutual interactions, depending on location. For this study, we adopt a generalized engineering system comprising n components or units.

When considering multiple hazards, three main features arise: (a) their **occurrence frequency**, (b) their **severity**, and (c) their **interdependency**. Collectively, these attributes encapsulate what is meant by "heterogeneous hazards," and we treat them as if they exist **independently of geographic context**—though in real life, they certainly vary by region. The first two (frequency and severity) are considered intrinsic hazard properties, modeled respectively by (a) a **non-homogeneous Poisson process** for their arrival times, and (b) a random distribution for severity. Detailed explanations of these models are deferred to Section 3. As in real practice, hazard interdependencies are divided into two classes: (c1) an **accelerating effect** on occurrence (i.e., one hazard can hasten the occurrence of another) and (c2) an

aggravating effect on severity (i.e., one hazard makes another more severe). We also quantify these effects in Section 3.

Each component's performance degrades over time under exposure to whichever hazards affect it, with the rate of degradation tied to both hazard frequency and severity (also elaborated in Section 3). It should be noted that each unit can have multiple performance indicators—each one matching up with a specific hazard. For example, **cyberattacks** may target server operations, consume system memory, and ultimately paralyze data services, whereas **hurricanes** cause physical damage by tearing down wires or weakening structural integrity. The deterioration of these performance indicators is what drives component failures, giving rise to a "competing failure" scenario—whichever hazard first pushes a performance indicator to its failure threshold effectively brings the component down. Once a failure occurs, the component undergoes an intervention or repair phase. During that period, the component is taken offline and restored to its baseline performance levels across all indicators. As a result, each component repeatedly cycles through hazard exposure and recovery phases, so its **availability** alternates between "online" (operational) and "offline" (unavailable) states over the long term. This cyclic behavior captures the idea of recurrent hazard impacts followed by repair, modeling how real-world systems maintain or lose functionality in response to successive hazardous events (Figure 1).

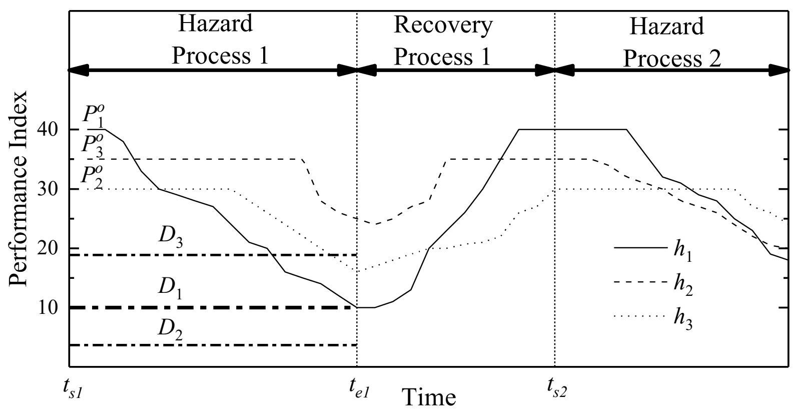

Figure 1. An example of component's behavior

Three kinds of hazards h_1, h_2, h_3 are involved, with their initial levels at P_1^o, P_2^o, P_3^o and failure thresholds at D_1, D_2, D_3, respectively. With the impact of hazards, the component has undergone two hazard process, of which (t_{s1}, t_{e1}) is finished as component fails at the first hit of h_1 to D_1; $(t_{s2},)$ is ongoing due to the hazards' non-arrival. Besides, the recovery process (t_{e1}, t_{s2}) manifest that, at the very moment t_{e1}, mitigation measures are conducted to resume the indexes of both three hazards and the component remain unavailable simultaneously until all indexes reached their desired level at t_{s2}. We introduce the following notations for the proposed models:

h_i :	the i^{th} type of hazard
$P_i(t)$:	the performance index of h_i at time t
P_i^o :	the initial performance index of h_i
$\vec{T}_i, T_{ik}$:	the occurrence time vector of h_i / the k^{th} occurrence time of h_i, $\vec{T}_i = (T_{ik}, k = 1, 2, \cdots)$
$\vec{S}_i, S_{ik}$:	the severity vector of h_i / the severity of k^{th} occurrence of h_i, $\vec{S}_i = (S_{ik}, k = 1, 2, \cdots)$
$\#\{\vec{T}_i\} / \#\{\vec{S}_i\}$:	the number of elements in vector $\vec{T}_i / \vec{S}_i$
$\lambda_i^o(t)$:	the intrinsic Poisson coefficient of h_i at time t
$S_i^o(t)$:	the intrinsic severity of h_i at time t
$\lambda_i(t)$:	the Poisson coefficient of h_i at time t
$f_{S_i^o(t)}(s)$:	the pdf that $S_{i,o} = s$ at time t

$\boldsymbol{H}_i^{fre}$:	the set of hazards that will impact on the occurrence frequency of h_i
$\boldsymbol{H}_i^{sev}$:	the set of hazards that will impact on the severity of h_i
$\tau_{i.j}^{fre} / \tau_{i.j}^{sev}$:	the effective period of the occurrence frequency/severity effect that h_i impacts on h_j
$\lambda_{i,j}(s_i)$:	the Poisson rate coefficient of h_j affected by h_i with severity s_i
$S_{i,j}(s_i)$:	the severity that h_j affected by h_i with severity s_i
$f_{\lambda_{i,j}(s_i)}(\lambda) / f_{S_{i,j}(s_i)}(s)$:	the pdf that $\lambda_{i,j}(s_i)=\lambda / S_{i,j}(s_i)=s$
D_i :	the failure threshold of h_i
$\alpha_{i,h}(t), \beta_{i.h} / \alpha_{i,r}(t), \beta_{i,r}$:	the parameter of hazard/recovery process of h_i at time t
$\alpha_{i.r}^{o}$:	the manual recovery rate of h_i
$\boldsymbol{\Omega}_m / N_m$:	the set of hazards applicable to the m^{th} node/the number of elements in $\boldsymbol{\Omega}_m$
$\boldsymbol{P}_{O\_S}(0,T)$:	the joint probability of all applicable $\vec{T}_i$ and $\vec{S}_i$
$P_{h\_r}(t_s' \mid t_s)$:	the conditional probability that a hazard-recovery process ends at t_s' given that it starts at t_s
$\Re_m(t)$:	the m^{th} node's reliability at time t
$P(t)$:	the performance of system at time t
Nm_t :	the normalized time at time t
$R(t)$:	the resilience of system at time t
τ_D	the time threshold for performance stage division
δ_D	the performance threshold for performance stage division
$\vec{T}_{k,w}=(t_{k,s}, t_{k,s}+1, \cdots, t_{k,e})$:	the time sequence of k^{th} stage
Tr_k :	the trend of the k^{th} stage

4 System Reliability Analysis

In this study, we use reliability metrics as performance indicators for probabilistic assessments of engineering systems faced with multiple external hazards. However, until now, little quantitative attention has been given to analyzing how multi-component systems, each with distinct characteristics, maintain reliability over an extended period when exposed to multiple, interdependent hazards. To address this gap, we formulate a framework that encompasses: (1) modeling multiple hazards and their interdependencies, (2) examining both hazard-induced degradation and subsequent recovery processes, and (3) deriving reliability measures for components and the overall system at arbitrary points in time.

4.1 Modeling of multiple hazards for each component

For m^{th} unit, we approach each of the hazards in $\boldsymbol{\Omega}_m$ from two perspectives: occurrence frequency and severity with the consideration of hazards' interdependence. The time vector $\vec{T}_i=(T_{i1}, T_{i2}, \cdots)$ is deployed to identify the occurrence time of the hazards *i*, (i.e., h_i) while the related $\vec{S}_i=(S_{i1}, S_{i2}, \cdots)$ is the mapping severity vector denoting the corresponding severity of h_i. It is worth unraveling that though 'severity' is a broad concept, here we designate severity as the degree of damage/threat to the component and thus, adopt the unit-free $\vec{S}_i$ to make a general reference. (e.g., the magnitude/intensity for earthquakes, the precipitation for rainstorms).

4.1.1 Hazards' Occurrence with Interdependence

In this work, the occurrence of h_i is assumed to follow a non-homogeneous Poisson counting process (NPP) with independent but non-stationary increment, characterized by a counting process $N_i(t) \sim \boldsymbol{P}\left(\int_0^t \lambda_i(s)ds\right)$. Apart from hazards' inherent properties, the hazards' interdependence is an influential to the term $\lambda_i(s)$; examples of this are rainstorm and flood may increase the occurrence probability of landslide/mudslide within a period, the drought and earthquake to the fire accident. We introduce the subset $\boldsymbol{H}_i^{fre} \in \boldsymbol{\Omega}_m$ to comprises the hazards that will give impetus to the occurrence of h_i (i.e., increase $\lambda_i(\square)$). Specifically, for an arbitrary hazard in $\boldsymbol{H}_i^{fre}$ (say, hazard *j*), its augmentation on $\lambda_i(t)$ is realize in terms of:

(i) $t \in \left(t_{jk} + \tau_{j,i}^{fre}, t_{jk}\right), \exists t_{jk} \in \vec{T}_j$, that is, t lies in the effective period of Hazard *j*'s k^{th} occurrence;

(ii) the number of all t_{jk} satisfying the constraint(i);

(iii) the severity S_{jk} w.r.t all t_{jk} which satisfying the constraint(i).

Therefore, given all $\vec{T}_j, \vec{S}_j, j \in H_i^{fre}$, $\lambda_i(t)$ can be derived as $\lambda_i(t) = \lambda_i^o(t) + \sum_{h_j \in \boldsymbol{H}_i^{fre}} \sum_{\forall t_{jk} \in \vec{T}_j ; t \in (t_{jk}, t_{jk} + \tau_{j,i}^{fre})} \lambda_{j,i}(S_{jk})$, where the $\lambda_i^o(t)$ indicates the inherent λ of hazard *i* at time *t* (particularly, to those secondary hazards $\lambda_i^o\left(t\right) = 0$), $\lambda_{j,i}(S_{jk})$ is the increment of $\lambda_i(t)$ contributed by the *k*th occurrence of h_j .

4.1.2 Hazards' Severity with Interdependence

Analogous, hazards' severity, as the other element, is decided jointly by hazards' inherent natures and their interdependence; the interactions on severity can be exemplified by that rainstorm may aggravate the impending flood and mudslide. Again, another subset $\boldsymbol{H}_i^{sev}$ is adopted for those will affect the severity of h_i (notably, $\boldsymbol{H}_i^{sev}$ and $\boldsymbol{H}_i^{fre}$ are not mutually exclusive). Particularly, for hazard *j* in $\boldsymbol{H}_i^{sev}$, its exacerbation on k^{th} occurrence of S_i (i.e., S_{ik}) is realized in terms of:

(i) $t_{ik} \in \left(t_{jl} + \tau_{j,i}^{sev}, t_{jl}\right), \exists t_{jl} \in \vec{T}_j$, t_{ik} lies in the effective period of Hazard *j*'s l^{th} occurrence;

(ii) the number of all t_{jl} satisfying the constraint(i);

(iii) the severity S_{jl} w.r.t all t_{jl} which satisfying the constraint(i).

Incorporating hazards interaction, we recursively derived the joint probability $\boldsymbol{P}_{O\_S}(0,T)$ as the product of the conditional probability of each hazard's behavior (i.e., $\vec{T}_i$ and $\vec{S}_i$) given its related hazards' (those in $\boldsymbol{H}_i^{fre}, \boldsymbol{H}_i^{sev}$) behavior:

$$\boldsymbol{P}_{O\_S}(0,T) = \prod_{h_i \in \boldsymbol{\Omega}_m} P\left(\vec{T}_i = \vec{t}_i, \vec{S}_i = \vec{s}_i \,\middle|\, \bigcup_{h_j \in \left(\boldsymbol{H}_i^{fre} \cup \boldsymbol{H}_i^{sev}\right)} \vec{T}_j = \vec{t}_j, \vec{S}_j = \vec{s}_j, h_j \in \boldsymbol{\Omega}_m\right)$$

4.2 Modeling of hazard/recovery process for each component

As explained in problem set, the recovery measures will be conducted once the hazards deactivated the component in a competing failure mode and intuitively, the followed hazards will retard the restoration (e.g., a rainstorm will hinder the assembling of wires, re-strike cyberattacks may impede the server reconstruction). Therefore, it can be conceivably hypothesized that the hazards' severity is embodied in 1) enhancing the degradation rate in hazard process 2) diminishing the repair rate in recovery process. For the hazard process (say, (t_s, t_e)), we deploy the monotonic Gamma process $X_{i,h}(t; \alpha_{i,h}(t), \beta_{i.h})$ to denote the degradation path for hazard i with time-varying $\alpha_{i,h}(t)$.

$$X_{i,h}(t_s) = P_i^o$$

$$\lim_{\Delta t \to 0} X_{i,h}(t + \Delta t; \alpha_{i,h}(t), \beta_{i,h}) = X_{i,h}(t; \alpha_{i,h}(t), \beta_{i,h}) - Ga(\alpha_{i,h}(t)\Delta t, \beta_{i,h}), t \in (t_s, t_e)$$

where the $Ga(\alpha_{i,h}(t)\Delta t, \beta_{i,h})$ denotes the gamma distribution with shape parameter $\alpha_{i,h}(t)\Delta t$ and scale parameter $\beta_{i,h}$. The $\alpha_{i,h}(t)$ is the piecewise function equaling the sum of the severity of all h_i 's occurrence in (t_s, t) :

$$\alpha_{i,h}(t) = \sum_{\forall T_{ik} \in \vec{T}_i, T_{ik} \in (t_s, t)} S_{ik}, t \in (t_e, t_s)$$

Referring to the recovery strategy, the recovery action will subsequently commence at t_e , for the recovery process (say, (t_e, t_s')), another Gamma process $X_{i,r}(t; \alpha_{i,r}(t), \beta_{i.r})$ is adopted; with the term $\alpha_{i,r}(t)$ indicates the recovery rate retarded by the follow-up hazards:

$$\lim_{\Delta t \to 0} X_{i,r}(t + \Delta t; \alpha_{i,r}(t), \beta_{i.r}) = X_{i,r}(t; \alpha_{i,r}(t), \beta_{i,r}) + Ga(\alpha_{i,r}(t)\Delta t, \beta_{i,r}), t \in (t_e, t_s')$$

$$X_{i,r}(t_s') = P_i^o$$
$$\alpha_{i,r}(t) = \alpha_{i,r}^o - \sum_{\forall T_{ik} \in \vec{T}_i, T_{ik} \in (t_e, t)} S_{ik}, t \in (t_e, t_s')$$

where $\alpha_{i,r}^o$ is the manual repair rate for h_i.

Next, we investigate the probability of one hazard-recovery process ends at t_s' given it starts at t_s, which can be decomposed in terms of failure inducement:

$$P_{h\_r}(t_s' \mid t_s, \vec{T}_i, \vec{S}_i, \forall h_i \in \mathbf{\Omega}_m) = \sum_{h_i \in \mathbf{\Omega}_m} P_{h\_r}^i(t_s' \mid t_s, \vec{T}_i, \vec{S}_i, \forall h_i \in \mathbf{\Omega}_m)$$

where the superscript i of term $\sum_{h_i \in \mathbf{\Omega}_m} P_{h\_r}^i(t_s' \mid t_s)$ indicates that the unit fails due to h_i in this hazard-recovery process and the explicit form of $P_{h\_r}^i(t_s' \mid t_s)$ is given by

$$P_{h\_r}^i\begin{pmatrix} t_s' \mid t_s, \vec{T}_i, \vec{S}_i \\ \forall h_i \in \mathbf{\Omega}_m \end{pmatrix} = \int_{t_s}^{t_s'} \underbrace{\int \cdots \int_0^{D_j}}_{\forall j \neq i, h_j \in \Omega_m} \begin{pmatrix} \prod_{\forall j \neq i, h_j \in \Omega_m} P(P_j(t_e) = p_j) \times P(P_i(t_e) = D_i)) \times \\ \sum_{\forall h_z \in \Omega_m} \Big(P(Inf(t \mid t > t_e, P_y(t) = P_y^o) < t_s') \times P(P_z(t_s') = P_z^o)) \Big) \end{pmatrix} d \cdots dp_j dt_e$$

Detailed forms of the terms in the bracket, which are determined by the degradation and recovery processes, can be furthermore decomposed as follows.

$$P(P_j(t_e) = p_j(t_e)) = f_{X_{j,h}}^{FHT}(p_j - P_j^o, \alpha_{j,h}(t), \beta_{j,h})$$
$$P(P_i(t_e) = D_i)) = f_{X_{i,h}}^{FHT}(D_i - P_i^o, \alpha_{i,h}(t), \beta_{i,h})$$
$$P(Inf(t \mid t > t_e, P_y(t) = P_y^o) < t_s') = \left(1 - F^{Ga}(P_y^o - P_y(t_e), \int_{t_e}^{t_s'} a_{y,r}(t)dt, \beta_{y,r}\right)$$
$$P(P_z(t_s') = P_z^o)) = f_{X_{z,r}}^{FHT}(P_z^o - P_z(t_e), \alpha_{z,r}(t), \beta_{z,r})$$

where the $f_{X_{i,h}/X_{i,r}}^{FHT}(x; a(t), b)$ indicates the pdf of the first hitting time to x that $X_{i,h} / X_{i,r}$ starting at t_s / t_e with shape parameter $a(t)$ and scale parameter *b*; $F^{Ga}(x; a, b)$ is the cdf of Gamma distribution.

4.3 Component Reliability Modeling

As the relative independency of each hazard-recovery process mentioned previously, no matter how much hazard-recovery process have the unit *m* experienced before, the reliability at an arbitrary time t only depends on the end of last hazard-recovery process t_{su} over $(0, t)$ as since t_{su}: 1) if all the performance index haven't hit their threshold until *t*, the units remains available and situates in hazard process at *t* 2)any performance index strikes its threshold and triggers the recovery process before *t*, the units is unavailable under recovery process. Therefore, the reliability at *t* can be obtained through investigating whether the unit is in hazard process or recovery process:

$$\Re_m(t) = \int_{\forall(\vec{T}_i, \vec{S}_i), h_i \in \mathbf{\Omega}_m} \sum_u \int_{t_{s2}} \cdots \int_{t_{su}} \begin{matrix} \boldsymbol{P}_{O\_S}(0,t) \times \prod_{n=1}^{u-1} P_{h\_r}\left(t_{sn}, t_{s(n+1)} \mid \vec{T}_i, \vec{S}_i, \forall h_i \in \mathbf{\Omega}_m\right) \\ \times P_{haz}(t_{eu} > t \mid t_{su}, \vec{T}_i, \vec{S}_i, \forall h_i \in \mathbf{\Omega}_m) \end{matrix} dt_{s2} dt_{s3} \cdots dt_{su} d(\vec{T}_i, \vec{S}_i)$$

As illustrated in section 3.1 and 3.2, the $\boldsymbol{P}_{O\_S}(0,t)$ and $P_{h\_r}\left(t_{sn}, t_{s(n+1)} \mid \vec{T}_i, \vec{S}_i, \forall h_i \in \mathbf{\Omega}_m\right)$ in term I indicates the joint probability of all $\vec{T}_i$ and $\vec{S}_i$ over $(0, t)$, and the probability that the hazard-recovery process ends $t_{s(n+1)}$ at given it starts at t_{sn} (note that $t_{s1} = 0$). The term J reflects the probability that component works under hazard process at t given the last hazard-recovery process ends at t_{su}, which can be further derived by investigating the probability all performance indexes have not exceeded the failure thresholds at t :

$$P_{haz}(t_{em} > t \mid t_{sm}, \vec{T}_i, \vec{S}_i, \forall h_i \in \mathbf{\Omega}_m) = \prod_{h_i \in \mathbf{\Omega}_m} P(P_i(t_s) < D_i)) = \prod_{h_i \in \mathbf{\Omega}_m} F^{Ga}(D_i - P_i^o, \int_{t_{sm}}^{t} \alpha_{i,h}(s)ds, \beta_{i,h})$$

4.4 The Reliability of system at Time *t*

After all units' reliability over time is calculated through above steps, one can obtain the system's reliability through structure functions (e.g., the well-known Newton-Raphson Method for the 3-machine 9-node system):

$$\Re(t) = SEM(\Re_1(t), \Re_2(t), \cdots)$$

The methodology and steps of most structure functions are standardized and is not the focus of this study, we therefore do not list the details in this study.

5. Simulation Verifications

We develop a simulation model to validate the accuracy of the proposed availability models. Specifically, we simulate the system availability over period $(0,T)$ under M dependent random hazards, where the distributions of hazards' frequency and severity are given. For comprehensiveness, we conduct a large number of simulation replications and take the average of the simulation results as the simulated availability. The simulated availability is adopted as the benchmark for verifying the availability obtained by the proposed theoretical models. The steps in one simulation replication are presented as follows.

Step 1: Allocate the M hazards into the following categorizes. Category 1 has M_1 hazards whose frequency and severity are not affected by other hazards. Category 2 has M_2 hazards whose frequency and severity are only affected by some of the hazards in Category 1. Following such a pattern, Category s has M_s hazards whose frequency and severity are only affected by the hazards in Categories $1,2,\cdots,s-1$.

Step 2: Generate hazards' occurrence time and severity.

Step 2*a*: Generate the occurrence time of the hazards $1,\ldots,M_1$ in Category 1, by generating M_1 vectors of random numbers $\vec{t}_1,\vec{t}_2,\cdots,\vec{t}_{M_1}$ that respectively follow $NPP(\lambda_1^0(t)), NPP\left(\lambda_2^0(t)\right),\cdots,NPP\left(\lambda_{M_1}^0(t)\right)$ for $t\in(0,T)$. Then, generate the severity of the M_1 hazards by generating M_1 vectors of random numbers $\vec{s}_1,\vec{s}_2,\cdots,\vec{s}_{M_1}$, where the M_1 vectors respectively have $\left|\vec{t}_1\right|,\left|\vec{t}_2\right|,\cdots,\left|\vec{t}_{M_1}\right|$ elements and each element in $\left|\vec{t}_i\right|$ ($i=1,2,\cdots,M_1$) follows the distribution with pdf $f_i^0(\cdot)$.

Step 2*b*: Obtain that $\lambda_i(t)=\lambda_i^0(t)+\sum_{j\in\Omega_i^{fre}}\sum_{\forall\beta:t_{j\beta}\in(t_{i\alpha}-\tau_{ji}^{fre},t_{i\alpha})}\lambda_i(S_{j\beta})$, $i=M_1+1,M_1+2,\cdots,M_1+M_2$. Generate the occurrence time of hazards $M_1+1,M_1+2,\cdots,M_1+M_2$ in Category 2, by generating M_2 vectors of random numbers $\vec{t}_{M_1+1},\vec{t}_{M_1+2},\cdots,\vec{t}_{M_1+M_2}$ that respectively follow $NPP(\lambda_{M_1+1}(t)), NPP(\lambda_{M_1+2}(t)),\cdots,NPP(\lambda_{M_1+M_2}(t))$ for $t\in(0,T)$ as in category 2. Then, generate the severity of hazards $M_1+1,M_1+2,\cdots,M_1+M_2$ by generating M_2 vectors of random numbers $\vec{s}_{M_1+1},\vec{s}_{M_1+2},\cdots,\vec{s}_{M_1+M_2}$, where the M_2 vectors respectively have $\left|\vec{t}_{M_1+1}\right|,\left|\vec{t}_{M_1+2}\right|,\cdots,\left|\vec{t}_{M_1+M_2}\right|$ elements and each element in $\vec{s}_i$ follows a distribution with pdf $f_i^0(\cdot)*\prod*_{\forall j\in\Omega_i^{Sev}}\prod*_{\forall\beta:t_{j\beta}\in(t_{i\alpha}-\tau_{ji}^{sev},t_{i\alpha})}f_{ij}\left(\cdot;S_{j\beta}\right)$.

Step 2*c*: Repeat step 2*b* for the hazards in Category $3,\cdots,s$ to simulate the occurrence time and severity of all hazards.

Step 3: Set initial inputs $t=0=t_{(1)}^1$, $I(0)=1$, $A_{Simu}(0)=1$ (where $A_{Simu}(t)$ is the simulated availability at time t), $D_i(0)=D_i^0$ for $i=1,\ldots,M$.

Step 4: Simulate system performance degradation and recovery processes.

Step 4*a*: Set $t=t+1$ and calculate $D_i(t+1)=D_i(t)+G\left(\sum_{\forall\alpha:\max\left\{t_{(\cdot)}^1\right\}<t_{i\alpha}\le t}s_{i\alpha},\beta_i\right)$ for $i=1,\ldots,M$. Move to step 5.

Step 4*b*: Set $t=t+1$ and calculate $D_i(t+1)=D_i(t)-N\left(v_i-\sum_{\forall\alpha:\max\left\{t_{(\cdot)}^1\right\}<t_{i\alpha}\le t}s_{i\alpha},\sigma_i\right)$ for $i=1,\ldots,M$. Move to step 5.

Step 5: Record $A_{Simu}\left(t+1\right)=1$ if $D_i(t)<D_i^F$ for $i=1,...,M$, record $t+1=t^{1}_{(\square)}$ if $A_{Simu}\left(t+1\right)=1$ and $A_{simu}\left(t\right)=0$, move to step 4*a*.

Record $A_{Simu}\left(t+1\right)=0$ otherwise, record $t+1=t^{2}_{(\square)}$ if $A_{Simu}\left(t+1\right)=0$ and $A_{simu}\left(t\right)=1$, move to step 4*b*.

Step 6: Stop at $t=T$.

As such, we obtain the value of $A_{Simu}(t)$ in one simulation replication for each $t\in\left(0,T\right)$. The six steps are replicated and the average of $\overline{A}_{Simu}\left(t\right)$ is referred to as the "simulated availability" at time *t*. A numerical example of the simulation is given as follows.

6. A Numerical Illustration

We consider a system that is subject to three dependent competing hazards. The hazards' inherent occurrence frequencies and severities respectively follow Poisson processes and normal distributions (Table 2). Among the three hazards, hazards 1 and 2 are mutually independent while hazard 3 is a secondary hazard which can only be triggered and amplified by hazard 2. The time windows within which the occurrence of hazard 2 has impacts on the occurrence rate and severity of hazard 3 are given in Table 3. The impacts of hazard 2 on the occurrence frequency and severity of hazard 3 are also given in Table 3. We use three Gamma processes and three Wiener processes to respectively model system performance degradation processes and recovery processes under the three hazards. The shape parameter of the Gamma process (i.e., $i=1,2,3$), the drift and diffusion parameters of the Wiener process (i.e., γ_i and σ_i^2, $i=1,2,3$), and system performance failure thresholds with respect to the three hazards $D_i^F, i=1,2,3$, are given in Table 4.

Table 2. Hazards' inherent frequency and severity

$\lambda_1^0(t)$	$\lambda_2^0(t)$	$f_1^0\left(\cdot\right)$	$f_2^0\left(\cdot\right)$	τ_1	τ_2	τ_3
0.005	0.007	$N(1.2,0.1)$	$N(1,0.1)$	50	50	50

Table 3. The parameter with respect to hazards' dependency

τ_{23}^{fre}	τ_{23}^{sev}	$\lambda_3(S_2)$	$f_{23}\left(\cdot;S_2\right)$
50	50	$0.005S_2$	$N(0.5,0.1)+S_2$

Table 4. The parameters of system performance degradation and recovery processes with respect to each hazard

<table>
<tr><th>β_1</th><th>β_2</th><th>β_3</th><th>γ_1</th><th>γ_2</th><th>γ_3</th><th>σ_1^2</th><th>σ_2^2</th><th>σ_3^2</th><th>D_1^F</th><th>D_2^F</th><th>D_3^F</th></tr>
<tr><td>5</td><td>5</td><td>10</td><td>20</td><td>15</td><td>25</td><td>16</td><td>16</td><td>16</td><td colspan="3">200</td></tr>
</table>

We conduct 50000 replications of simulation over period $\left(0,100\right)$ and take the average of the simulation results as the simulated availability. The simulated availability and the availability obtained by the theoretical models are plotted in Figure 2, where the two availabilities are approximately the same over the analysis period. The difference between the simulated availability and the availability obtained by the proposed theoretical model is induced by the approximation in (i) truncating the number of state transitions during the simulation period, which indeed, is infinite and (ii) discretizing the hazards' occurrence time and severity, which are indeed, continuous.

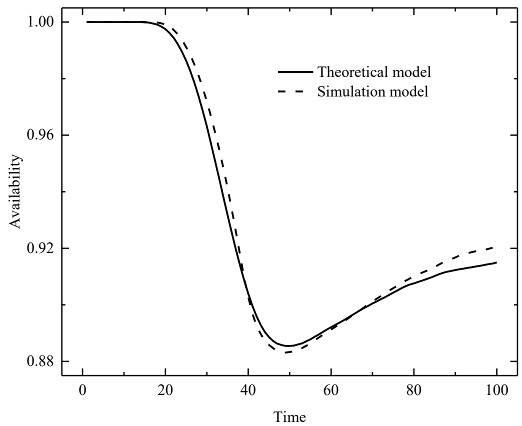

Figure 2. System availability obtained by the proposed model and the simulation model

We can observe that the curves produced by the simulation model and the theoretical model are quite close, which demonstrates the feasibility of the proposed model.

7. Conclusions

In this paper, we analytically investigate the performance (i.e., availability) and resilience of systems under dependent random hazards with recurrence. First, we quantitatively characterize hazards' occurrence frequency and severity, considering that their occurrence frequency (and severity) can be increased (and amplified) by their dependent hazards. We obtain the joint distribution of the occurrence time and severity of all hazards that pose threats to the system. Based on the hazards characterization, we investigate system performance degradation and recovery processes under the hazards and accordingly, obtain system multimodal availability across time. We then employ availability as system overall performance and model the instantaneous resilience of the system. The accuracy and efficiency of the proposed method are verified by a simulation study and a numerical case.

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