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An Optimal Joint Sampling, Inspection and Maintenance Plans for One-Shot Units

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Abstract

Many one-shot units are often stored for extended periods before their single use, during which they may experience on-shelf degradation and failure. To ensure these units perform reliably when needed, timely inspections and maintenance are essential. However, inspecting and maintaining every stored unit in full detail can be economically prohibitive. In this study, we propose a joint sampling, inspection and maintenance (SIM) plans tailored for heterogeneous one-shot units. Rather than examining every unit, our approach involves inspecting a selected sample to assess overall performance, followed by an imperfect maintenance process. The optimal SIM plan seeks to balance the availability of reliable one-shot units with the efficient use of SIM resources under various scenarios. This is achieved by determining (i) the criteria for selecting inspection samples and (ii) the appropriate maintenance and restoration thresholds for the products. A numerical example based on a type of airbag is provided to demonstrate the effectiveness of the proposed SIM plans, along with recommendations for the SIM scenarios best suited to different circumstances.

Keywords
Sampling, Inspection, Maintenance, One-Shot Units

1. Introduction

One-shot products, such as airbags and sealed lead acid batteries, are often stored for long durations before their sole intended use (Cheng and Elsayed, 2018, Cheng and Elsayed, 2017). Over time, these items can experience gradual deterioration—stemming from factors like aging, chemical reactions, corrosion, and thermal fatigue—which may impair their performance when they are finally deployed. For instance, a sealed lead acid battery might slowly self-discharge and suffer corrosion at its metal contacts during storage, thereby reducing its effectiveness. Likewise, safety devices held in standby, such as airbags, are vulnerable to environmental stresses. In one notable case, moisture infiltrated an airbag's inflator during storage, inducing thermal fluctuations that weakened its structural integrity and ultimately led to a deployment that released hazardous metal fragments.

To ensure that these one-shot units perform as required when activated, engineers advocate for periodic performance evaluations and maintenance throughout the storage period. However, given the continuous production and accumulation of these items, it is neither practical nor cost-effective to examine every unit in exhaustive detail at every interval. This challenge has led to the development of dynamic, sampling-based joint sampling, inspection, and maintenance (SIM) strategies. These flexible SIM plans focus on assessing a representative subset of units to infer the overall condition of the stored inventory while judiciously deploying maintenance resources. Such an approach aims to maintain the high reliability of the entire collection of one-shot units without incurring prohibitive inspection and maintenance costs.

Existing SIM studies fall short when addressing populations of one-shot units that are stored for prolonged periods. First, these units are produced and added to storage continuously, which means the overall inventory comprises items of varying ages. Consequently, the extended storage period and staggered production require inspections and maintenance actions to be conducted repeatedly—and not in a uniform, one-size-fits-all manner. Moreover, given the resource constraints and the vast number of units, it is impractical to perform a full, detailed inspection and maintenance on every single item during each review cycle. Instead, our approach involves sampling a subset of units at each inspection to evaluate their overall condition and to carry out maintenance as needed. Notably, the samples selected in successive inspections do not have to be identical; a unit might be inspected during one cycle and then again later, while others might only be checked once or not at all. This results in a storage population where units have experienced different inspection frequencies and maintenance histories.

Furthermore, the degradation process during storage varies among units due to inherent differences in material properties and manufacturing variability, leading to diverse maintenance outcomes. These variations in unit age, inspection frequency, and maintenance history necessitate a comprehensive evaluation of the overall availability and reliability of the stored inventory. Additionally, gathering operational reliability data throughout the storage period requires a hybrid SIM plan that leverages various inspection techniques to capture different aspects of unit performance.

In this work, we develop a sequence of hybrid sampling-based SIM strategies specifically for heterogeneous one-shot units that undergo long-term storage. We analyze the overall availability of these units, taking into account the costs associated with inspections, maintenance actions, and potential failures. Based on this analysis, we determine two key elements at the outset of the evaluation period: (i) the characteristics and size of the inspection sample, which are updated at each inspection, and (ii) the maintenance and restoration thresholds for the units, which are set as fixed values. Numerical validation of the proposed SIM plan supports its effectiveness, and we subsequently provide recommendations for selecting the most appropriate SIM strategy for products exhibiting varying failure rates, inspection frequencies, and inspection durations.

The remainder of this paper is organized as follows. In Section 2, we review the current state-of-the-art in the field. Section 3 give the problem statement. Section 4 details and solve the optimal SIM plans. Section 5 presents numerical examples that illustrate the application of the IM plans, and finally, Section 6 concludes the paper.

2. Literature Review, Gap and Contributions

One shot units are widely deployed in many fields of modern engineering (Chen and Elsayed, 2017, Cheng and Elsayed, 2016). Products may fail as a result of intrinsic degradation, external shocks, or a combination thereof. In general, intrinsic failure mechanisms are modeled using stochastic processes such as Gamma, Inverse Gaussian (IG), or Cox models, whereas external shocks are typically represented by either homogeneous or non-homogeneous Poisson processes. These shock processes can induce either sudden catastrophic failures or a gradual deterioration in performance. In many cases, these failure mechanisms interact and compete with one another (Zhang et al., 2021, Zhang et al., 2016, Peng et al., 2019). For instance, Zhang et al. (2021) examined a system exposed to two distinct failure types: one governed by a three-phase degradation process (normal, defective, and failed) and another triggered by random shocks. In their work, the durations of transitions between degradation stages are characterized by specific probability distributions, and shocks arriving via a Poisson process may lead to system failure with a given probability. To reduce overall costs, a fixed-interval inspection policy combined with a hybrid corrective maintenance (CM) and preventive maintenance (PM) strategy was implemented. Similarly, other studies have employed a Wiener process with stage-specific parameters to model multi-stage degradation (Zhang et al., 2016) or considered shock-induced increments in degradation in conjunction with imperfect inspections (Peng et al., 2019). Rafiee et al. (2015) focused on degradation driven by multiple shock types and subsequently proposed a hybrid CM-PM policy that incorporates imperfect maintenance, while Liu et al. (2016) extended this approach by developing a CM-PM policy for systems experiencing several degradation processes, where the effectiveness of PM diminishes with successive interventions and replacement is triggered when degradation exceeds a predefined threshold. Additional research by Zheng et al. (2016) and Zhu et al. (2016) has addressed systems with both repairable and irreparable failures, including those comprising components vulnerable to either abrupt hard failures or progressive degradation. In contrast, Wu and Castro (2020) modeled a system in which degradation is depicted as a weighted combination of various processes, with routine PM and system renewal following multiple maintenance actions. Moreover, progressive IM strategies—where maintenance and subsequent inspection decisions are made based on the most recent inspection outcomes—have also been explored (Li and Pham, 2005).

Furthermore, SIM strategies have been developed for systems with interdependent failure modes. Researchers employing models such as the Cox model (Zheng and Makis, 2020, Tang et al., 2015, Hu et al., 2020, Caballé et al., 2015) and stochastic frailty models (Liu et al., 2013) have devised approaches for systems in which (i) the instantaneous failure rate is influenced by the current degradation level (Zheng and Makis, 2020), (ii) the system becomes increasingly susceptible to shocks once degradation exceeds a critical threshold (Caballé et al., 2015), or (iii) the failure rate is affected by both the system's age and its degradation status (Tang et al., 2015, Liu et al., 2013). Chen et al. (2012) imposed an upper limit on the number of PM actions and designed a hybrid maintenance strategy for systems featuring both maintainable and non-maintainable failure modes, where the maintainable failure rate is modulated by the cumulative incidence of non-maintainable failures. Moreover, interdependencies among failure modes can be bidirectional; for instance, the degradation level may be correlated with the shock arrival rate (Yousefi et al., 2020, Huynh et al., 2011), or the frequencies of different failure modes may mutually influence one another (Fan et al., 2011). External factors, such as the severity of shocks (Peng et al., 2010, Keedy and Feng, 2013, Cha et al., 2016) and environmental conditions (Zhu et al., 2015), can further induce or modify these dependencies. Yang et al. (2018) introduced a dynamic maintenance limit to adaptively schedule inspection and replacement intervals for systems whose failure rates and degradation levels are jointly affected by external shocks. Zhao et al. (2025) investigated a state-specific maintenance and inventory policy for a system subject to self-announcing failures.

Population heterogeneity is generally attributed to the intrinsic characteristics of units, such as differences in failure rates. Finkelstein (Finkelstein, 2004) proposed a minimal maintenance policy for a mixed population consisting of several subgroups with distinct failure rates, a framework later extended with statistical inference in (Cha and Finkelstein, 2011). In a similar vein, Xiang et al. (2013) optimized key parameters—including the burn-in threshold, burn-in interval, and age-based replacement interval—for populations characterized by different lifetime distributions, and subsequently validated the efficiency of their proposed SIM policy by comparing it with two alternative approaches (Xiang et al., 2014). Moreover, several studies (Cha and Finkelstein, 2012, Zhang et al., 2014, Van Oosterom et al., 2017, Cheng et al., 2022, Cheng et al., 2021, Cheng and Elsayed, 2021) have developed SIM strategies for scenarios in which a unit is randomly selected from a population divided into two heterogeneous subpopulations, typically representing normal and defective units. Cha and Finkelstein (2012) examined the effect of preventive maintenance on the reliability of such units, while (Wei and Cheng, 2025),(Wei et al., 2025) and (Wei et al., 2024) investigated the optimal SIM strategies for a fleet of self-service systems. A comparable approach is proposed in (Zhang et al., 2014), where a condition-based replacement policy is employed during early life, transitioning to an age-based strategy later to prevent wear-out failures. Additionally, Van Oosterom et al. (2017) employed a Markov decision process to schedule replacements for a deteriorating unit by selecting a spare from a heterogeneous population characterized by a variable state transition probability matrix.

In summary, we can identify several gaps in the existing literature on SIM plan design. First, current SIM strategies for products with multiple failure mechanisms have primarily concentrated on single-unit systems, treating all failure modes in aggregate. Typically, the optimal SIM approach is derived based on the overall system reliability across all failure modes, following an "inspect-all, maintain-all" paradigm that fails to distinguish the individual impacts of each failure mode, thereby leading to suboptimal allocation of maintenance resources. Furthermore, existing SIM plans are generally either fixed in pattern—employing constant inspection intervals and maintenance thresholds—or are updated only when real-time inspection data become available. Second, current SIM plans for heterogeneous populations can be broadly classified into two categories. The first category focuses on single-unit-based strategies, wherein a unit (or its spare) is randomly selected from a population that exhibits intrinsic variability in failure or degradation behaviors (e.g., normal versus defective subpopulations). The second category addresses the entire heterogeneous population under the assumption of a single failure mode, typically without employing any sampling procedure.

In this work, we address these gaps by developing a sequence of hybrid sampling-based IM plans tailored for heterogeneous one-shot units. Specifically, we optimize the availability of these units, along with the costs associated with inspections, maintenance actions, and failures, by determining the optimal number of units to sample and establishing the appropriate preventive maintenance threshold for systems subject to degradation-induced failure.

3. Problem Statement

The one-shot units are sequentially produced in batches and stored in a warehouse prior to delivery to users. We consider that n_i units are produced in the i^{th} batch and stored at time w_i. The product experiences *N* independent competing failure modes throughout the storage period, out of which M failure modes lead to sudden failures while the other (*N-M*) modes are associated with product's degradation with observable indicators. As a product's reliability metrics randomly differ from one unit to another, we include such a randomness into product's failure and degradation processes characterization by the following statistical models:

(i) The lifetime of the p^{th} sudden failure mode ($p=1,...,M$) follows a Weibull distribution with cumulative distribution function (CDF) $F_p(t)=1-\exp\left(-(t/\theta_p)^{\lambda_p}\right)$ where θ_p and λ_p are the scale and shape parameter, respectively;

(ii) The p^{th} degradation mode $\left(p=M+1,\ldots,N\right)$ is represented by a degradation process (Ye et al., 2013) $D_p\left(t\right)=D_{0\_p}\cdot\left(1+\beta_p\cdot t\right)+\varepsilon_p$, where D_{0_p} is the corresponding initial degradation level, β_p is a normally-distributed coefficient with $\beta_p\sim N\left(\mu_{\beta_p},\sigma_{\beta_p}^2\right)$ and ε_p is a normally-distributed measurement error with $\varepsilon_p\sim N\left(\mu_{\varepsilon_p},\sigma_{\varepsilon_p}^2\right)$. The probability that the performance degradation with respect to the p^{th} failure mode crosses the failure threshold D_p^F by time t can be expressed as $P\left(D_{0\_p}\cdot\left(1+\beta_p\cdot t\right)+\varepsilon_p\geq D_p^F\right)=1-\Phi\left((D_p^F-D_{0\_p}-\mu_{\varepsilon_p}-\mu_{\beta_p}t)/\sqrt{\sigma_{\beta_p}^2\cdot t^2+\sigma_{\varepsilon_p}^2}\right)$. For a specific p, a constant D_p^F is employed throughout $\left(0,T\right)$.

It is evident that a product can fail either when a sudden failure occurs or when its degradation level in a specific mode reaches the designated failure threshold. Next, the problem statement regarding the SIM scenario is given as follows.

Inspection Procedure

Non-destructive inspections (NDTs) are conducted repeatedly in a non-identical pattern. In the j^{th} inspection, *N* NDTs start simultaneously at time t_j to respectively inspect the product's performance with respect to the *N* failure modes. In particular, *s* sample of $\sum_{\forall i}\sum_{\forall \mathbf{I}:j\in\mathbf{I}_p}n_{i,\mathbf{I}}$ units (with heterogeneous characteristics) is selected and tested with respect to their performance on the p^{th} failure mode (p = 1,…, N) in a non-destructive pattern. It is worth pointing out that different samples of units are selected for different NDTs and one unit may be inspected for multiple failure modes in the j^{th} inspection and similarly, inspected for the same failure mode in multiple inspections. Units are sampled with an equal probability. The inspection time for the p^{th} failure mode is predetermined as τ_p, which is independent of the sample size. The cost to inspect a unit's performance with respect to the p^{th} failure mode in one inspection is a constant value $C_{p\_\text{NDT}}^{\text{insp}}$. The unit remains unavailable during the inspection.

Maintenance Procedure

A preventive maintenance (PM) action on the one-shot unit's p^{th} failure mode is immediately performed following the inspection if the unit's deterioration level exceeds a certain threshold. If a unit fails due to p^{th} sudden failure ($p=1,\ldots,M$) upon the inspection, a corrective maintenance (CM) is then executed to bring the one-shot unit's state back to a brand-new one. We assume that the duration of CM and PM follows the same distribution with CDF $F_P(r_p)$. Further, the unit cost to maintain a unit's performance with respect to the p^{th} failure mode is a constant value C_p^{main}. Note, C_p^{main} is given in terms of cost per time unit for $p=M+1,\ldots,N$. The total cost to maintain a unit's performance with respect to the p^{th} failure mode depends on D_p^R and $D_p(t_j)$, as derived later in section 4.

Following the above scheme, the IM condition of an individual unit by time T can be respectively indicated by vectors $\boldsymbol{I} = \left(\boldsymbol{I}_p, p = 1,2,\cdots,N\right)^T$ and $\boldsymbol{M} = \left(\boldsymbol{M}_p, p = 1,2,\cdots,N\right)^T$, where $\boldsymbol{I}_p = \left(\boldsymbol{I}_p^{(1)}, \boldsymbol{I}_p^{(2)}, \cdots\right)$ and $\boldsymbol{M}_p = \left(\boldsymbol{M}_p^{(1)}, \boldsymbol{M}_p^{(2)}, \cdots\right)$ are respectively composed of inspections in which the unit's performance with respect to the p^{th} failure mode is inspected and maintained. Note, $\boldsymbol{M}_p \subseteq \boldsymbol{I}_p$ always holds $\forall p$.

Figure 1 depicts the SIM condition corresponding to a unit's performance regarding the p^{th} failure, while Figure 2 illustrates the unit's overall performance in relation to its SIM condition. In these figures, the symbols Δ ,d , and denote, respectively, the following scenarios: no inspection is conducted for the p^{th} failure mode, an inspection is carried out without subsequent maintenance, and both inspection and maintenance are performed.

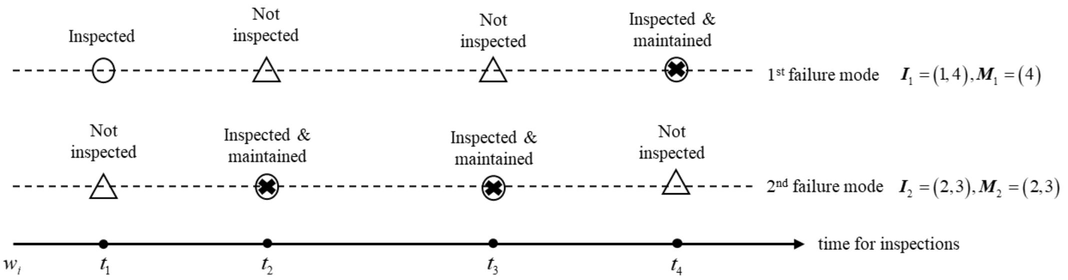

Figure 1. An Example of the One-Shot Units' SIM conditions

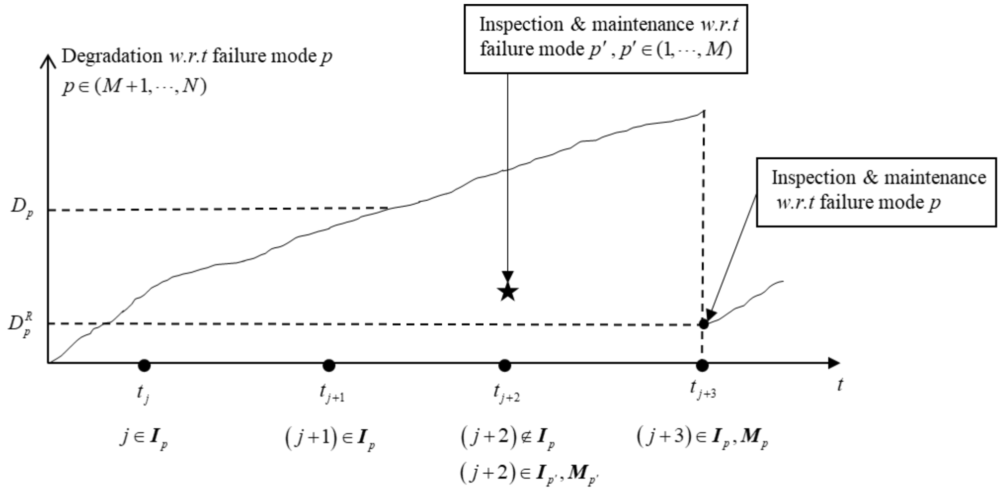

Figure 2. An Example of the One-Shot Units' Performance under different SIM conditions

4. Optimal SIM Plan Design

Optimal SIM plan design is crucial because it ensures the efficient use of resources while maximizing the reliability of one-shot units stored for extended periods. One-shot products, such as safety devices or batteries, experience degradation over time due to various factors, and improper functioning upon activation can create significant risks or operational failures. Therefore, the key objective of the optimal SIM plan is to identify potential failure modes and to strategically schedule inspections and maintenance actions in a manner that mitigates the risks while optimizing the allocation of limited inspection and maintenance resources. As these products are continuously produced and stored in large quantities, examining every unit in detail for all potential failure modes is neither practical nor cost-effective. The optimal design encourages an effective balance between inspection coverage and resource expenditure. By selecting representative samples for inspection and maintenance, manufacturers can assess the performance of the entire product population without exhausting available resources. This approach also allows for periodic maintenance and timely interventions, which collectively ensure that the product's quality and reliability are sustained throughout its storage period. An optimal SIM plan accounts for variations in material properties and manufacturing processes, tailoring actions to safeguard the overall reliability of the stored units, regardless of their age or previous maintenance history. In conclusion, without an optimal SIM plan, products may fail prematurely or inefficiently, undermining the goal of ensuring operational safety and effectiveness.

Based on the previous problem statement, we can formulate the optimization problem as:

Minimize
$$\sum_{\forall i}\sum_{\forall \boldsymbol{I}} n_{i,\boldsymbol{I}} \cdot \#\{\boldsymbol{I}\} \tag{1}$$

s.t.
$$A_{Overall}(0,T) \geq \overline{A}, C_{Overall}(0,T) \leq \overline{C}, n_{i,\boldsymbol{I}:\#\{\boldsymbol{I}\}=0} = 0,\ \forall i, \boldsymbol{I} \tag{2}$$

where the decision variables include: (i) the one-shot unit's maintenance threshold D_p^M ($p = M+1,\ldots,N$) and (ii) the sample size with respect to each failure mode in each inspection $n_{i,\boldsymbol{I}}, \forall i, \boldsymbol{I}$ in each SIM by time *T*.

For an arbitrary unit in the i^{th} batch, the probability that it is inspected under $\boldsymbol{I}_p, \forall p$ is $P_{i,\boldsymbol{I}} = \frac{n_{i,\boldsymbol{I}}}{n_i}$ as units are sampled with equal probability. Then, provided that a unit is inspected under $\boldsymbol{I}_p, \forall p$, the probability that it is maintained with respect to the *p*th failure mode $P_{i,\boldsymbol{M}|\boldsymbol{I}}$ can be derived conditionally as:

$$P_{i,\boldsymbol{M}|\boldsymbol{I},p=1,\ldots,M} = \prod_{\alpha=1,\ldots,\#\{\boldsymbol{M}_p\}} \left(F_p\left(t_{\boldsymbol{M}^p_{(\alpha)}} - t_{\boldsymbol{M}^p_{(\alpha-1)}} - r_p\right) - F_p\left(t_{\boldsymbol{I}^p_{(\alpha')}} - t_{\boldsymbol{M}_p(\alpha-1)} - r_p\right)\right) R_p\left(t_{\max\{\boldsymbol{I}_p\}} - t_{\max\{\boldsymbol{M}_p\}} - r_p\right) \tag{3}$$

$$P_{i,\boldsymbol{M}|\boldsymbol{I},p=M+1,\ldots,N} = \prod_{\alpha=1}^{\#\{\boldsymbol{M}_p\}} P\begin{pmatrix} D_p\left(t_{\boldsymbol{M}_p^{(\alpha)}} - t_{\boldsymbol{M}_p^{(\alpha-1)}} - r_p\right) \geq D_p^M \\ \left| D_p\left(t_{\boldsymbol{I}_p^{(\alpha')}} - t_{\boldsymbol{M}_p^{(\alpha-1)}} - r_p\right) < D_p^M \right. \end{pmatrix} \cdot D_p\left(t_{\max\{\boldsymbol{I}_p\}} - t_{\max\{\boldsymbol{M}_p\}} - r_p\right) < D_p^M \tag{4}$$

where $\boldsymbol{M}_p^{(\alpha'+1)} = \boldsymbol{I}_p^{(\alpha)}$. Note that in Eqs. (5) and (6), the sudden and degradation-induced failures are addressed separately. Specifically, for $p = 1,\ldots,M$ ($p = M+1,\ldots N$), the unit will be maintained in $\boldsymbol{M}_p$ with the probability that the failure occurs suddenly (its degradation level surpasses D_p^M) between $\left(t_{\boldsymbol{I}_p^{(\alpha)}} + \tau_p, t_{\boldsymbol{M}_p^{(1)}}\right)$ and $\left(t_{\boldsymbol{I}_p^{(\alpha')}} + \tau_p, t_{\boldsymbol{M}_p^{(\alpha)}}\right)$ for $\alpha = 1,\ldots,\#\{\boldsymbol{M}_p\}$.

A unit's overall availability is the product of its availability with respect to all the *N* failure modes, i.e., $A_{i,\boldsymbol{I},\boldsymbol{M}}(t) = \prod_{p=1}^{N} A_{i,\boldsymbol{I},\boldsymbol{M}}(t;p)$. We herein develop the availability (with respect to the *p*th failure mode) of a unit with characteristics $i, \boldsymbol{I}, \boldsymbol{M}$ as a piecewise function of its IM condition.

Specifically, a unit's survival probability between two consecutive inspections depends on the maintenance actions performed. If no maintenance is carried out after the latter inspection, the unit is guaranteed to survive—that is, it maintains a survival probability of 1 throughout the interval. On the other hand, if a maintenance action is executed following the latter inspection, the unit's survival probability becomes a time-dependent function that decreases monotonically over the interval. Moreover, this survival probability is further influenced by the maintenance decision made after the previous inspection: if no maintenance was conducted after the former inspection, the probability starts at 1 and steadily falls to 0 by the next inspection; alternatively, if maintenance was performed after the former inspection, the initial probability is reduced to a value between 0 and 1, which then similarly declines to 0 as time progresses. It is also important to note that the unit is unavailable for use during all inspection and maintenance periods.

Table 1 summarizes the availability of a unit with respect to a sudden failure mode. When assessing a unit's availability with respect to degradation modes, the conditional availability needs to be alternatively replaced by the probability that the unit's performance level with respect to the *p*th degradation mode does not reach D_p^F at time *t*, given it reaches (or does not reach) D_p^M in the inspection afterwards.

Table 1. The availability with respect to the *p*th failure mode ($p = 1,2,\ldots M$) of an unit with characteristics $i,\boldsymbol{I},\boldsymbol{M}$ during (0,*t*)

t	Conditions	Availability of a Unit with Characteristics $i,\boldsymbol{I},\boldsymbol{M}$
$t \in \left(w_i, t_{\boldsymbol{I}_p^{(1)}}\right)$	$\boldsymbol{I}_p^{(1)} < \boldsymbol{M}_p^{(1)}$	1
	$\boldsymbol{I}_p^{(1)} = \boldsymbol{M}_p^{(1)}$	$1-\frac{F_p(t-w_i)}{F_p\left(t_{\boldsymbol{I}_p^{(1)}}-w_i\right)}$
$t \in \left(t_{\boldsymbol{I}_p^{(1)}}, t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p\right)$; $\alpha = 1,\ldots,\#\{\boldsymbol{I}_p\}$	Always	0
$t \in \left(t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p, t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p+r_p\right)$; $\alpha = 1,\ldots,\#\{\boldsymbol{I}_p\}$	$\exists\alpha' : \boldsymbol{I}_p^{(\alpha)} = \boldsymbol{M}_p^{(\alpha')}$	0
	$\nexists\alpha' : \boldsymbol{I}_p^{(\alpha)} = \boldsymbol{M}_p^{(\alpha')}$ & $\exists\alpha'' : \boldsymbol{I}_p^{(\alpha+1)} = \boldsymbol{M}_p^{(\alpha'')}$	$1-\frac{F_P\left(t-t_{\boldsymbol{I}_p^{(\alpha')}}-\tau_p-r_p\right)}{F_P\left(t_{\boldsymbol{I}_p^{(\alpha+1)}}-t_{\boldsymbol{I}_p^{(\alpha')}}-\tau_p-r_p\right)}$, where $\boldsymbol{I}_p^{(\alpha'+1)} = \boldsymbol{M}_p^{(\alpha)}$
	$\nexists\alpha' : \boldsymbol{I}_p^{(\alpha)} = \boldsymbol{M}_p^{(\alpha')}$ & $\nexists\alpha'' : \boldsymbol{I}_p^{(\alpha+1)} = \boldsymbol{M}_p^{(\alpha'')}$	1
$t \in \left(t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p+r_p, t_{\boldsymbol{I}_p^{(\alpha+1)}}\right)$; $\alpha = 1,\ldots,\#\{\boldsymbol{I}_p\}-1$	$\exists\alpha'' : \boldsymbol{I}_p^{(\alpha+1)} = \boldsymbol{M}_p^{(\alpha'')}$	$1-\frac{F_p\left(t-t_{\boldsymbol{I}_p^{(\alpha)}}-\tau_p-r_p\right)}{F_p\left(t_{\boldsymbol{I}_p^{(\alpha+1)}}-t_{\boldsymbol{I}_p^{(\alpha)}}-\tau_p-r_p\right)}$
	$\nexists\alpha'' : \boldsymbol{I}_p^{(\alpha+1)} = \boldsymbol{M}_p^{(\alpha'')}$	1
$t \in \left(t_{\max\{\boldsymbol{I}_p\}}+\tau_p+r_p, T\right)$	Always	$\frac{1-F_P\left(t-t_{\max\{\boldsymbol{M}_p\}}-\tau_p-r_p\right)}{1-F_P\left(t_{\max\{\boldsymbol{I}_p\}}-t_{\max\{\boldsymbol{M}_p\}}-\tau_p-r_p\right)}$

Table 2 summarizes the availability of a unit with respect to a deterioration failure mode. For a unit with characteristics $i,\boldsymbol{I},\boldsymbol{M}$, its availability with respect to the *p*th failure mode ($p = M+1,\ldots,N$) during period (0,*T*) is derived as follows:

Table 2. Units' availability with respect to the *p*th failure mode ($p = M+1,\ldots,N$)

t	Conditions	Availability of a Unit with Characteristics $i,\boldsymbol{I},\boldsymbol{M}$
$t \in \left(w_i, t_{\boldsymbol{I}_p^{(1)}}\right)$	$\boldsymbol{I}_p^{(1)} < \boldsymbol{M}_p^{(1)}$	1
	$\boldsymbol{I}_p^{(1)} = \boldsymbol{M}_p^{(1)}$	$1-\frac{P\left(D_p(t-w_i) \ge D_p^F\right)}{P\left(D_p\left(t_{\boldsymbol{I}_p^{(1)}}-w_i\right) \ge D_p^M\right)}$
$t \in \left(t_{\boldsymbol{I}_p^{(1)}}, t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p\right)$; $\alpha = 1,\ldots,\#\{\boldsymbol{I}_p\}$	Always	0
$t \in \left(t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p, t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p+r_p\right)$; $\alpha = 1,\ldots,\#\{\boldsymbol{I}_p\}$	$\exists\alpha' : \boldsymbol{I}_p^{(\alpha)} = \boldsymbol{M}_p^{(\alpha')}$	0
	$\nexists\alpha' : \boldsymbol{I}_p^{(\alpha)} = \boldsymbol{M}_p^{(\alpha')}$ & $\exists\alpha'' : \boldsymbol{I}_p^{(\alpha+1)} = \boldsymbol{M}_p^{(\alpha'')}$	$1-\frac{P\left(D_P\left(t-t_{\boldsymbol{I}_p^{(\alpha')}}-\tau_p-r_p\right) > D_p^F\right)}{P\left(D_P\left(t_{\boldsymbol{I}_p^{(\alpha+1)}}-t_{\boldsymbol{I}_p^{(\alpha')}}-\tau_p-r_p\right) > D_p^M\right)}$

	$\nexists\alpha':\boldsymbol{I}_p^{(\alpha)}=\boldsymbol{M}_p^{(\alpha')}$ & $\nexists\alpha'':\boldsymbol{I}_p^{(\alpha+1)}=\boldsymbol{M}_p^{(\alpha'')}$	1
$t\in\left(t_{\boldsymbol{I}_p^{(\alpha)}}+\tau_p+r_p,t_{\boldsymbol{I}_p^{(\alpha+1)}}\right)$; $\alpha=1,\ldots,\#\left\{\boldsymbol{I}_p\right\}-1$	$\exists\alpha'':\boldsymbol{I}_p^{(\alpha+1)}=\boldsymbol{M}_p^{(\alpha'')}$	$1-\dfrac{P\left(D_P\left(t-t_{\boldsymbol{I}_p^{(\alpha'')}}-\tau_p-r_p\right)>D_p^F\right)}{P\left(D_P\left(t_{\boldsymbol{I}_p^{(\alpha+1)}}-t_{\boldsymbol{I}_p^{(\alpha'')}}-\tau_p-r_p\right)>D_p^M\right)}$
	$\nexists\alpha'':\boldsymbol{I}_p^{(\alpha+1)}=\boldsymbol{M}_p^{(\alpha'')}$	1
$t\in\left(t_{\max\left\{\boldsymbol{I}_p\right\}}+\tau_p+r_p,T\right)$	Always	$\dfrac{P\left(D_p\left(t-t_{\max\left\{\boldsymbol{M}_p\right\}}-\tau_p-r_p\right)<D_p^F\right)}{P\left(D_p\left(t_{\max\left\{\boldsymbol{I}_p\right\}}-t_{\max\left\{\boldsymbol{M}_p\right\}}-\tau_p-r_p\right)<D_p^M\right)}$

5. Case Study

In this section, we numerically illustrate the optimal inspection and maintenance plan design for a type of airbag by considering its production and retrieval details. Specifically, the airbag production process is organized into three distinct batches. The first batch, produced at time 0, consists of 5 units; the second batch, produced at time 500, comprises 7 units; and the third batch, produced at time 1000, contains 10 units. These production times and retrieval sizes are integral to the scheduling framework of the optimal IM plan.

The following two failure modes are related to sudden failures and degradation of such airbags. The first failure mode is a sudden failure mode (failure mode 1), and the failure time follows the Weibull distribution with CDF $F_1(t)=1-\exp\left(-(t/\theta_1)^{1.5}\right),\theta_1=6000$, the second failure mode is a degradation-induced failure mode (failure mode 2) where the degradation path is modeled by $D_2\left(t\right)=0.65\cdot\left(1+t\right)$ where the failure threshold is $D_2^F=75$.

Other parameters are given in Tables 3 and 4 as follows.

Table 3. Costs associated with IM of the airbags, airbag's failures and disposal

p^{th} failure mode	$C_{p\_\text{NDT}}^{\text{insp}}$	$C_{p\_\text{DT}}^{\text{insp}}$	C_p^{main}	C^{fail}	C^{salv}
1	20	35	5	90	300
2	30	50	5	110	

Table 4. The inspection time and inspection duration of the airbags

Inspection Time					
t_1	t_2	t_3	t_4	τ_1	τ_2
600	1100	1600	1950	5	7

Based on the model proposed in this paper, we employed a genetic algorithm to determine the optimal solution under the constraint the up to 400 monetary units can be used for the sampling and inspection actions. The optimal solutions are given as follows:

- When no SIM actions are implemented, the average unit availability is 1030.6.
- When the number of units inspected for a specific failure mode is kept constant across all inspections, the optimal solution indicates that the preventive maintenance threshold for the second failure mode is 41. Under this scenario, 4 samples are drawn during each inspection for failure mode 1 and 5 samples for failure mode 2, resulting in a total availability of 1368.

- When the sampling sizes can vary across inspections, the optimal solution yields a preventive maintenance threshold for the second failure mode of 53. Specifically, 1, 3, and 5 samples are drawn in three successive inspections for failure mode 1, while 3, 5, and 6 samples are drawn in three successive inspections for failure mode 2, leading to a total availability of 1432.

These results demonstrate that the SIM approach proposed in this study effectively balances sampling thoroughness across different inspection actions and optimizes resource allocation, thereby enhancing the average unit availability of one-shot units over a specified period. Furthermore, we observe that when sampling sizes are allowed to vary across inspections, the optimal preventive maintenance thresholds are higher, and the average availability of the one-shot units is correspondingly increased. This finding indicates that the proposed maintenance policy enables a more precise and strategically nuanced control of the maintenance process, thereby effectively enhancing system performance under variable inspection conditions.

5. Conclusions

In this study, we propose and develop a series of optimal SIM plans for one-shot units that are stored for extended periods and subjected to competing failure modes. The inherent heterogeneity of these units—reflected in variations in age, inspection history, and maintenance conditions—arises from continuous production and the repeated execution of SIM actions, thereby complicating the design of effective maintenance strategies. The proposed SIM plans are formulated to optimally allocate limited inspection and maintenance resources (for example, by minimizing the total number of inspections at an acceptable cost) while ensuring that the reliability metrics of the one-shot units are maintained at a desired level. Our numerical analysis investigates performance of the optimal SIM plans and its superiority over other alternatives.

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