Proceedings of the International Conference on Industrial Engineering and Operations Management

Publisher: IEOM Society International, USA
Published: February 18, 2025

DOI: 10.46254/AN15.20250078

Developing Mathematical Model for Dynamic Energy-Aware Scheduling of Diffusion Furnaces with Machine Eligibility and Waiting Time Restrictions

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Abstract

The semiconductor manufacturing industry, inherently capital and technology-intensive, faces growing complexities in production scheduling due to increasing process sophistication and stringent energy requirements. Diffusion Furnaces (DFs), which are essential in semiconductor doping processes, are critical bottlenecks due to their long processing times, high energy consumption, and the need to accommodate incompatible job families with specific time constraints. This study proposes a dynamic scheduling model for DFs aimed at minimizing total costs, including penalties for tardiness, time window violations, and electricity usage under a Time-of-Use (TOU) tariff structure. A mathematical model is formulated for non-identical parallel DFs, incorporating machine eligibility restriction (MER). The proposed mathematical model is empirically validated using randomly generated small size numerical problem and the LINGO solver. The results obtained from LINGO Solver for the numerical problem confirms the correctness of the model proposed for the research problem considered in this study. While the proposed model achieves optimal total cost, they exhibit computational intractability for large-scale problems. This research contributes to the advancement of efficient scheduling strategies for DFs in semiconductor manufacturing, with potential for significant cost reductions and improvements in energy efficiency.

Keywords
Semiconductor manufacturing, Diffusion Furnaces, Incompatible Job-Families, Machine Eligibility Restriction, Time Window, Energy-Aware, Mathematical Model.

1. Introduction
The semiconductor industry is highly capital- and technology-intensive, with manufacturers continuously innovating to keep pace with Moore's Law. As transistor density increases, production complexity rises, requiring efficient scheduling to minimize costs, energy consumption, and delays. This study focuses on scheduling diffusion furnaces (DFs), a critical bottleneck in wafer fabrication. DFs, essential for doping, operate as parallel batching machines with long processing times, contributing to 30% of total work-in-process (WIP). Given rising demands, minimizing tardiness is crucial to maintaining efficiency and customer satisfaction.

Semiconductor manufacturing is energy-intensive (Schorn et al.,2023), with DFs consuming significant electricity due to high operating temperatures of 973K–1523K (Tan et al., 2020). Optimizing energy use not only reduces costs but also enhances sustainability by lowering greenhouse gas emissions. Implementing Time-of-Use (TOU) pricing strategies—scheduling high-energy operations during off-peak hours—can significantly cut electricity expenses and improve grid stability. Beyond energy savings, adhering to time window constraints is essential, as delays degrade job quality and may result in scrapped wafers, increasing costs and jeopardizing timely deliveries (Jung et al.,2014). Given the high WIP levels at DFs, efficient scheduling that balances tardiness penalties, energy costs, and time constraints is key to improving operational performance in semiconductor fabs.

The paper is organized as follows: The closely related review is presented in section 3 and the problem statement with assumptions in scheduling of diffusion furnace is presented in section 3. In section 4, a mathematical model is proposed and demonstrated its workability using a suitable numerical example. Finally, a summary of contributions along with some possible future research directions are discussed in section 5.

2. Literature Review

(Ikura and Gimple, 1986) are recognized pioneers in Batch Production Manufacturing (BPM) scheduling. In semiconductor manufacturing (SM), (Mathirajan and Sivakumar, 2006) noted that BPM scheduling, particularly for Diffusion Furnaces (DFs), has been widely studied from a deterministic perspective. With computerized job tracking, deterministic scheduling is well-supported. (Vimala and Mathirajan, 2023) provided a comprehensive review of DF scheduling, classifying it into static and dynamic approaches. While static scheduling assumes all jobs are available at the start, dynamic scheduling accounts for jobs arriving at different times (Uzsoy, 1995), leading to improved performance. Solution methodologies are categorized into mathematical programming-based and heuristic-based approaches (Vimala and Mathirajan, 2023).

Several studies have applied mathematical programming to optimize DF scheduling. (Dobson and Nambimadom, 2001) addressed batching with incompatible job families, minimizing mean weighted flow time. (Su, 2003) examined a two-stage flow shop with batch and single processors under hard time window constraints. (Artigues et al., 2006) developed a Decision Support System using the disjunctive graph method for multiple performance measures. (Monch and Almeder, 2009) extended DF scheduling to a parallel machine setting, minimizing total weighted tardiness. (Vimalarani, 2017) introduced real-time events and machine eligibility constraints, making dynamic scheduling more applicable to real-world scenarios.

Recent studies integrate energy efficiency into scheduling models. (Rocholl et al., 2020) incorporated time-of-use electricity pricing, formulating a bi-criteria scheduling model using the ε-constraint method to balance total weighted tardiness and energy consumption. (Jung et al., 2014) emphasized time window constraints to prevent chemical degradation, ensuring jobs enter the DF within a specified timeframe to maintain product quality and avoid job scrapping. This study builds on previous research by developing a cost-based objective function that incorporates tardiness cost, time window violation cost, and electricity cost, utilizing a mathematical programming-based approach.

3. Problem description and assumption:

We are considering **N** jobs and **M** machines, represented with **F in-compatible job families** for scheduling to prevent cross-contamination. And each job, **j** of a family **f** (f = 1, 2, …, F) has processing time of **PTF_f**, due-date of **DJ_j**, and **non-zero** release time of **RJ_j** (at which it becomes available for processing), the release time and due-dates are **non-agreeable** (that is, if $RJ_i \leq RJ_j$ not-implied $DJ_i \leq DJ_j$). Some jobs also have a **time-window TW_j**, requiring processing within a specific period after release to avoid rework or scrapping. Jobs with compatible processing requirements are batched together. The batch start time in each machine *m*, is the maximum of latest release time of jobs in that batch and the machine available time. **No pre-emption** is allowed. Processing incurs electricity costs, governed by **Time-of-Use (TOU) tariffs**, where prices vary across three periods: **off-peak (low cost), intermediate, and peak (high cost)**. Batches may span multiple periods, leading to varying electricity costs. The task here is to schedule the batches with respect to above mentioned constraints within the **time horizon** of **[0, T],** where 0 is considered as the start time and T is the make span of processing all batches divided by total number of machines in the NPDF. It is assumed that T is greater than sum of processing time of all the batches implying a feasible solution.

For the research problem described above, the objective of the research is to ***minimise total cost (TC).*** It can be defined as the sum of penalty cost incurred due to tardy job , penalty cost of time window violation and electricity cost. [that is, $Total\ cost = \sum_{j=1}^{N}\left(TP_j * TJ_j\right) + \sum_{j=1}^{N}\left(TWVP_j * TWV_j\right) + \sum_{m=1}^{L}\sum_{p=1}^{P}\sum_{b=1}^{K}\left(APTB_{pbm} * EC_p * PUB_b\right)$. The **tardiness** of job *j* can be defined as how much late the job *j* is completed as compared to the due-date, i.e., positive lateness [$TJ_j = \max(0, CJ_j - DJ_j)$, where TJ_j, CJ_j and DJ_j are tardiness, completion time and due-date of job *j*, respectively]. Jobs can be with or without time window constraint and the one which come with **time window constraint** must enter the DF within a specified time from it release time, i.e., [$TWV_j = \max(0, RB_b - (RJ_j + TW_j)$, where RJj, TW_j, TWVj, $TWVP_j$ and RB_b are the time window, release time, time window violation, unit penalty for violation of the job j and release time of the batch respectively] violation of time window constraint may affect the quality of the output. Additionally, to this we are calculating the **energy consumption** in terms of electricity cost using Time of Use (TOU) pricing policy where $APTB_{pbm}$, EC_p, PUB_b assigned processing time of the batch on a machine in a particular period, electricity price for period p, and power consumption per batch, respectively.

These objective balances customer satisfaction (minimizing tardiness), product quality (time window adherence), and cost-efficiency (electricity savings), aligning with sustainability goals. Using the (Graham et al., 1979) three-field notation, the scheduling problem is defined as: *"1/p-batch, dynamic job-arrivals, time window constraint, considering the occurrence of real time events, in-compatible job families, due-dates, release time, non-agreeable release times and due-dates , energy consumption/Total cost"* and we make the following assumptions:

- All data related to DS of a NPDF problem considered in this study are assumed to be deterministic and known a priori. In practice, estimates of the required parameters' values for this problem can be obtained from the existing shop-floor computerised information system, which tracks work-in-process inventory and the machine status in real time.
- Real-time events related to jobs and resources will occur randomly at any given time.
- Each job requires one operation, and all jobs are independent.
- The each NPDF has a capacity CF_m, measured in terms of number of jobs it can hold. The number of jobs in a batch must be less than or equal to CF_m. All jobs in a batch must be of the same family.
- Once processing of a batch is initiated, it cannot be interrupted, and other jobs cannot be introduced into the any of the NPDF until processing of the batch is completed.
- The time window constraint of the jobs is known in prior. Jobs belonging to same family have same time window.
- In case of violation, it would be subject to soft penalty. The jobs which don't have time window to enter the diffusion furnace will not face any penalty.
- All the batches are completed within the time horizon, T, where is greater than the sum of all processing time of the batches implying feasible solution.
- The time horizon T for each NPDF is divided into three periods, each with a different cost per unit time. The electricity cost is calculated by multiplying the cost per unit of each period by the hours of processing in that period, and then by the units consumed per unit time of processing for the given batch.

4. Proposed Mathematical Model for cost minimization in DS-NPDF with MER:

Objectives in scheduling can be categorized into completion time-based and due date-based (Vimalarani et al., 2022). Rocholl et al. (2020) proposed a mixed integer linear programming (MILP) model to minimize total weighted tardiness and electricity costs using a TOU policy for parallel batch machines. Elaoud et al. (2024) combined MILP with heuristic and decomposition techniques to optimize KPIs such as cycle time and time link violations. Vimalarani and Mathirajan (2015) developed (0-1) MILP models to minimize total weighted tardiness (TWT).

This study employs a (0-1) Mixed integer non-linear programming (MINLP) model to optimize total cost, incorporating tardiness, time window violation, and electricity costs. Unlike previous studies, it considers multiple diffusion furnaces (DFs) with varying capacities and machine eligibility restrictions (MER). The proposed (0-1) MINLP models address (a) DS-NPDF and (b) DS-NPDF with MER. This section details the model formulation, validation, and computational complexities, followed by nomenclature, constraints, and mathematical models.

A nomenclature

Sets:

J	Jobs
F	Families
B	Batches
P	Periods
M	Machines (Diffusion furnaces)

Index:

j	1…N for jobs
b	1…K for batches
f	1…G for families

p	1…*P* for periods
m	1…L for machines

Parameters:

FT_m	First-time availability of the machine '*m*'
CF_m	Batch capacity of machine '*m*'
AFM_{fm}	Assigned family to the machine [That is, when $AFM_{fm} = 1$ indicates that the family '*f*' can be processed on machine '*m*' and = 0 indicates that the family can't be processed in the concerned machine '*m*' to family '*f*'.]
RJ_j	Release time of a job '*j*'
DJ_j	Due date of a job '*j*'
TWJ_j	Time window of job '*j*'
PTF_f	Processing time of a family '*f*'
PUB_b	Units of power consumed per unit time by a batch '*b*' in period '*p*'.
FJ_{jf}	Family association for a job, [That is, when $FJ_{jf} = 1$ indicates that the job '*j*' belongs to family '*f*' and = 0 indicates that the family doesn't belong to family '*f*'.]
SP_p	Start time of period '*p*'.
EP_p	End time of period '*p*'.
EC_p	Electricity cost of period '*p*'.

Decision variables:

JB_{jbm}	**1** if job '*j*' is processed in a batch '*b*' in a machine '*m*'; **0** Otherwise.
FB_{fbm}	**1** if family '*f*' is processed in a batch '*b*' in a machine '*m*'; **0** Otherwise.
BP_{pbm}	**1** if batch '*b*' is processed in a machine '*m*' in period '*p*'; **0** Otherwise.
$Z1_{pbm}$	**1** if batch '*b*' processing time spans in single period '*p*' ; **0** is if batch '*b*' processing time spans across two periods

Dependent variables:

CB_{bm}	Completion time of a batch '*b*' in a machine '*m*'
RB_{bm}	Release time of a batch '*b*' in a machine '*m*'
PB_{bm}	Processing time of a batch '*b*' in a machine '*m*'
SJ_j	Start time of job '*j*' w.r.t RB_{bm}
CJ_j	Completion time of job '*j*' w.r.t CB_{bm}
TJ_j	Tardiness of job '*j*' w.r.t CJ_j
TP_j	Unit tardiness penalty of job '*j*' per unit time (that is, per hour)
$TWRJ_j$	Time window of job '*j*' w.r.t release time of the job '*j*'
TWV_j	Time window violation of job '*j*'
$TWVP_j$	Time window violation penalty of job '*j*' per unit time (that is, per hour)
$APTB_{pbm}$	Assigned processing time of batch '*b*' in a machine '*m*' in period '*p*'.

Formulation of (0-1) MINLP Model for DS of NPDF to minimise Total cost.

Minimise

$$\sum_{j=1}^{N}\left(TP_j * TJ_j\right) + \sum_{j=1}^{N}\left(TWVP_j * TWV_j\right) + \sum_{m=1}^{L}\sum_{p=1}^{P}\sum_{b=1}^{K}\left(APTB_{pbm} * EC_p * PUB_b\right)$$

Subject to

$$\sum_{f=1}^{G} FB_{fbm} \le 1 \qquad \forall b \in [1,K] \tag{2}$$

$$\sum_{f=1}^{G} FB_{f,b-1m} \ge \sum_{f=1}^{G} FB_{fbm} \qquad \forall b \in [2,K] \tag{3}$$

$$\left(JB_{jbm} * FJ_{jf}\right) \le FB_{fbm} \qquad \forall m \in [1,L]; \forall j \in [1,N]; \ \forall b \in [1,K]\,; \forall f[1,G] \tag{4}$$

$$\sum_{j=1}^{N}(JB_{jbm} * FJ_{if}) \ge FB_{fbm} \qquad \forall m \in [1,L]\,; \forall f \in [1,G]; \ \forall b \in [1,K] \tag{5}$$

$$FBM_{fbm} \le AFM_{fm} \qquad \forall m \in [1,L] \forall b \in [1,K]\,; \forall f[1,G] \tag{6}$$

$$\sum_{b=1}^{K}\sum_{m=1}^{L} JB_{jbm} = 1 \qquad \forall m \in [1,L]; \ \forall j \in [1,N] \tag{7}$$

$$\sum_{j=1}^{N} JB_{jb} \leq CF_m \qquad \forall b \in [1, K] \tag{8}$$

$$RB_{bm} \geq FT_{bm} \qquad b = 1\,; \forall m \in [1, L] \tag{9}$$

$$RB_{bm} \geq \left(RJ_j * JB_{jbm}\right) \qquad \forall m \in [1, L]; \forall j \in [1, N];\ \forall b \in [1, K] \tag{10}$$

$$RB_{bm} \geq CB_{b-1,m} \qquad \forall b \in [2, K]; \forall m \in [1, L] \tag{11}$$

$$PB_{bm} = \sum_{f=1}^{G} PTF_f * FB_{fbm} \qquad \forall m \in [1, L];\ \forall b \in [1, K] \tag{12}$$

$$CB_{bm} = \ RB_{bm} + PB_{bm} \qquad \forall m \in [1, L]; \forall b \in [1, K] \tag{13}$$

$$CJ_j \geq CB_{bm} - (BigM) * \left(1 - JB_{jbm}\right) \ \forall m \in [1, L]; \forall b \in [1, K]; \in [1, N] \tag{14}$$

$$SJ_j \geq \ RB_{bm} - (BigM) * \left(1 - JB_{jbm}\right) \forall m \in [1, L]; \forall b \in [1, K];\ \forall j \in [1, N] \tag{15}$$

$$TJ_j \geq \ CJ_j - DJ_j \qquad \forall j \in [1, N] \tag{16}$$

$$TWRJ_j \geq \ RJ_j + TW_j \qquad \forall j \in [1, N] \quad s.t.\ TW_j > 0 \tag{17}$$

$$TWV_j \geq \ SJ_j - TWRJ_j \qquad \forall j \in [1, N] \tag{18}$$

$$\sum_{p=1}^{P} APTB_{pbm} = PB_{bm} \qquad \forall m \in [1, L];\ \forall b \in [1, K] \tag{19}$$

$$\textstyle\sum_{b=1}^{K} APTB_{p1bm} \ \leq EP_{p1m} - RB_{b1m} \qquad \forall m \in [1, L] \tag{20a}$$

$$\sum_{b=1}^{K} ASPT \ \leq EP_{pm} - SP_{pm} \qquad \forall m \in [1, L]; \forall p > 1 \tag{20b}$$

$$ASPT \ \leq BS_{pbm} * BP_{pbm} * \ PB_{bm} + \left(1 - BS_{pbm}\right) * BigM \tag{21a}$$

$$APTB_{pbm} \ \geq (1 - BS_{pbm}) * BP_{pbm} * \ \left(EP_{pm} - \ RB_{bm}\right) \tag{21b}$$

$$PB_{bm} - \left(EP_{pm} - \ RB_{bm}\right) \leq \left(1 - BS_{pbm}\right) * BigM \tag{21c}$$

$$\left(EP_{pm} - \ RB_{bm}\right) - \ PB_{bm} + 1 \leq BS_{pbm} \ * BigM \qquad \forall m[1, L]; \forall b[1, K];\ \forall p\ [1, P] \tag{21d}$$

$$BP_{pbm} = \begin{cases} 1, & (RB_{bm} \ \leq EP_{pm}) * (CB_{bm} \ \geq \ SP_{pm}) \\ 0, & otherwise \end{cases} \qquad \forall m \in [1, L]; \forall b \in [1, K];\ \forall p \in [1, P] \tag{22}$$

$$BS_{pbm} \in \{0,1\} \qquad \forall m \in [1, L]; \forall b \in [1, K]\,; \forall p \in [1, P] \tag{23}$$

$$JB_{jbm} \in \{0,1\} \qquad \forall m \in [1, L];\ \forall j \in [1, N];\ \forall b \in [1, K] \tag{24}$$

$$FB_{fbm} \in \{0,1\} \qquad \forall m \in [1, L];\ \forall f \in [1, G];\ \forall b \in [1, K] \tag{25}$$

The objective function presented in (1) is to minimize the total cost which includes tardiness penalty, time window violation penalty and electricity cost. Constraints (2) to (8) are related to batch formation for NPDF. Constraint (2) ensures that each batch can process jobs from only one family. A batch cannot mix jobs from different families. Constraint (3) is used to form the batches sequentially. Constraint (4) ensures that a job is assigned to a family batch only if it belongs to that family. Constraint (5) Ensures that a batch allocated to a family contains at least one job from that family. Constraint (6) ensure the machine eligibility restriction of the diffusion furnace. Constraint (7) ensures that each job is assigned to exactly one batch and one machine. Constraint (8) guarantees that the total number of jobs in a batch does not exceed the capacity of the machine. Constraints (9) to (11) are related to the release time of a batch. Constraint (9) guarantees that the release time of the first batch is not less than the first-time availability of the earliest available machine of NPDF. In addition to that, the release time of any batch should be greater than the maximum arrival time of jobs in that batch and the completion time of the previous batch. For the aforementioned constraints, Constraint (10) and Constraint (11) are formed. Constraint (12) is used to calculate the dependent variable processing time of batch. It can be calculated from the processing time of family that constitutes in that batch. Constraint (13) calculates the dependent variable completion time of batch. It is the sum of release time and processing time of batch. Constraint (14) is used to calculate the completion time of the job. This Ensures the job completion time is linked to the batch's completion time, using a large constant *BigM* to enforce the condition only when the job is assigned to the batch. Constraint (15) Ensures the job's start time is aligned with the batch's release time if the job is part of that batch. Constraint (16) is used to calculate the tardiness of jobs (that is, positive lateness). It can be calculated as the difference between the completion time of job and its due date.

Next set of constraints i.e. the constraint (17) and Constraint (18) are associated with time window constraint. Constraint (17) calculates the time window of job with respect to release time of the job. It is the sum of release time and time window of the job. Constraint (18) captures the time window violation of each job, which is the difference between the start time of the batch and the release time of the job with time window.

Constraint (19) to constraint (24) deal with calculating electricity cost using time of use (TOU) policy. Constraint (19) makes sure that the sum of assigned processing time of a batch in all periods is exactly equal to its processing time of the batch. Constraint (20) calculates that the sum of processing time assigned to a period should be less than or equal to the duration of the period. Constraints (21a to 21d) designed to assign the total time units spent by the batch in that period. Particularly constraint 21(a) ensures that if the batch fits within the period (when BS_{pb}=1), the assigned processing time $APTB_{pb}$ is less than or equal to the processing time of the batch PB_b. If the batch does not fit within the period (when BS_{pb}=0), this constraint does not impose any restriction due to the large value of BIG_M. (21b) constraint ensures that if the batch does not fit within the period (when BS_{pb}=0), the assigned processing time ASB_{pb} is at least the remaining time of the period $\left(EP_p - \ RB_b \right)$. Constraints (21c) and (21d) ensure that BS_{pb} is set correctly. If PB_b is less than or equal to the remaining time of the period ($EP_p - \ RB_b$), BS_{pb} is set to 1. Otherwise$BS_{pb}$is set to 0. Constraint (22) to (25) assign the binary values to the decision variables. All constraints are formulated with respect to machines, ensuring compatibility and scheduling feasibility across jobs, batches, and families on each machine.

4.2 Validation of the Proposed Mathematical Models for DS-NPDF with MER

To validate the proposed (0-1) MINLP model for DS-NPDF with MER, a numerical problem is formulated with two diffusion furnaces. The job characteristics, including job code, family ID, release time, due date, time window, power consumption, and penalty costs, are provided in a Table 1 and Table 2. The diffusion furnace capacity is measured in terms of the number of jobs it holds. In general, a diffusion furnace can accommodate between 6 and 12 standard jobs, but for this small-scale problem, it is assumed to hold 2 and 3 jobs, respectively. The initial availability of the furnaces for processing a batch is set at the first and second hours, respectively. Machine 2 is not eligible to process jobs from family 3. A LINGO set code is developed to generate and solve the (0-1) MINLP model for any dataset. The LINGO solver provides the optimal solution for the objective function, minimizing the total cost.

Table 1. A numerical problem to validate the proposed (0-1) MINLP model for DS-SDF with MER

Job Code	Family Identification	Release Time	Time window	Due date	Time window violation cost	Tardiness cost
J1	5	1	10	30	2	1
J2	1	5	-	60	-	6
J3	4	5	8	57	16	8
J4	3	6	4	50	20	10
J5	2	5	-	16	-	8
J6	2	2	-	3	-	6
J7	3	2	4	8	4	2
J8	5	6	10	21	20	10
J9	5	6	10	50	10	5
J10	5	1	10	12	2	1

Table 2. A summary of the additional data for the proposed experimental design for small-scale problems of DS-NPDF with MER

Parameters	**Number of Levels**	**Level wise Values**
Number of machines (DFs)	2	MC1, MC2
Capacity of the machines	2	[2,3] for machines MC1 and MC2 respectively.
Availability of the machines	2	1st and 2nd hours for MC1 and MC2 respectively.
Machine eligibility restriction	1	Machine 2 cannot process jobs from family 3

The objective function value obtained for DS-NPDF with MER (Machine Eligibility Restriction) for 10 jobs is **843.** The MER is applied to jobs belonging to family 3 which cannot be processed in Machine 2 . The summary of optimal solution presented in the below Table 3 shows the analysis (0-1) MINLP model representing the problem on DS-NPDF with MER.

Table 3. Optimal solution for the numerical example considered to validate the proposed (0-1) MINLP model for DS-NPDF with MER

Machine	**Batch**	**Job**	**ST_J**	**TW_J**	**$ST_J + TWJ$**	**RT_B**	**PT_B**	**CT_B**	**DD_J**	**$TWVC_J$**	**TC_J**	**TWV_J**	**T_J**	**UNITS**	**P1(ECp1=3)**	**P2(ECp2=2)**	**P3(ECp3=1)**	**$TTWVC_J$**	**TTC_J**	**TEC_B**
M1	**B1**	J2	3	-	-	3	2	5	10	-	3	3	0	3	2			-	0	18
	B2	J4	3	4	7	5	10	15	5	8	4	0	10	2	10			0	40	60
		J7	5	4	9	5	10	15	35	12	6	0	0					0	0	0
	B3	J3	2	8	10	15	16	31	7	14	7	5	24	1	9	7		70	168	41
M2	**B1**	J5	4	-	-	4	4	8	11	-	5	-	0	3	4			-	0	36
		J6	2	-	-	4	4	8	13	-	9	-	0					-	0	0
	B2	J1	4	10	14	8	20	28	29	8	4	0	0	2	16	4		0	0	112
		J9	6	10	16	8	20	28	15	16	8	0	13					0	104	0
		J10	6	10	16	8	20	28	20	12	6	0	8					0	48	0
	B3	J8	5	10	15	28	20	48	21	4	2	13	27	1		20		52	54	40
TOTAL																		122	414	307
																		843		

ST_J- Release time of job**; TW_J**- Time window of job; **RT_B**- Ready time of batch; **PT_B**-Processing time of batch; **CT_B**-Completion time of Batch; **DD_J**-Due-date of job; **$TWVC_J$**- Time window violation penalty of job; **TC_J**- Tardiness penalty of job; **TWV_J**- Time window violation; **T_J**-Tardiness; **UNITS**-Units of power consumed by a batch; **ECp1,ECp2,ECp3**- electricity cost per unit power per unit time in respective period; **$TTWVC_J$**- Total Time window violation penalty of job; **TTC_J**- Total Tardiness penalty of job; **TEC_B –** Total electricity cost.

5. Summary

In this study, the optimal solution for the research problem defined is sought through a mathematical modelling approach. Specifically, (0-1) mixed integer non-linear programming [(0-1) MINLP] model was developed for the Dynamic Scheduling (DS) of Non-Identical Parallel Diffusion Furnaces (NPDF) with Machine Eligibility Restrictions (MER), all aimed at minimizing the total cost. This (0-1) MINLP model was validated by (i) generating a representative numerical example and (ii) developing a corresponding LINGO code to solve the model using the LINGO solver. Given the complexity of large-scale batch scheduling, solely relying on mathematical models for dynamic scheduling decisions is impractical. Hence as a part of our future research we intend to develop and implement heuristic approaches that can generate efficient (optimal or near-optimal) solutions within a shorter computational time.

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